

## Author's response to the referees

I am grateful to the referees for attentive revision of the manuscript. I agree that the comments given by the referees are important and allow to improve the manuscript.

1) *I have numerous issues with the model descriptions and a lot of suspicions about its results. My major concern with the results is that the amplitudes of the calculated shelf deflections are larger than the amplitude of waves forcing these deflections. These results are physically inconsistent due to a very simple argument: to the zero's order the results of this model should be similar to results of the shelf treatment as a thin beam (e.g. Vaughan, 1995). Exact solution of the beam equation with periodic forcing (eqn (3) in Vaughan, 1995) shows that the deflection amplitude cannot exceed the amplitude of the forcing amplitude. Of course, the shelf deflection could be amplified due to resonance, however, it is difficult to believe that it was the case in all scenarios considered in this study. Most likely, there is a sign error in the implementation of the boundary condition on the shelf base.*

I agree with these comments. Indeed, both the model of Holdsworth & Glynn and Vaughan's model give the solutions for the periodic forcing in which the deflection amplitude doesn't exceed the amplitude of the incoming ocean wave forcing. Unfortunately, revision of the program code hasn't revealed sufficient errors in the code and, thus, the results presented in the manuscript haven't been changed significantly. The sign in the implementation of the boundary conditions on the shelf base stems from the basic boundary conditions  $\sigma_{ik} n_k = -P n_i$  that are included into the momentum equations. Certainly, in the case of a sign error in the boundary conditions we can vary signs directly in the program code and look to the changes in the solution to be sure that the errors lead to exceeding of the deflection amplitude above the forcing amplitude... I don't expect that errors in the program code still exist, but I can suggest too that the problem for the full equations is unstable. I mean that although the approach based on inclusion of the basic boundary conditions into the momentum equations provides stabilizing of the numerical solution, the instability of the problem reveals considerable variations in the solution (in the deflection amplitude) as response to variations of model parameters (ice shelf length, Young's modulus, Poisson's ratio)... Probably, we can obtain the stable solution in the case of the field equations, which contains Navier-Stokes equation coupling with continuity equation for the water flow under ice shelf, i.e. in the case of more general mathematical statement of the problem, in which viscous fluid under ice shelf is considered. Obvious advantage of the model based on the wave equation (Eq. (2) in Holdsworth & Glynn, 1978) is that the incoming water flux is not required as the boundary condition at  $x = L$ . Likely, the incoming water flux can be chosen from the condition of agreement between solutions obtained in the full model and in the model of Holdsworth & Glynn respectively.

2) *With regards to the model formulation, it is difficult to follow its description due to mixed presentation of the governing equations and numerical implementation. It will be beneficial to present the governing equations in their strong form without referencing to the numerical implementation. If the author finds useful to include a description of numerics, it should be done separately. It seems that there are typos in the right hand sides of all boundary conditions (apart from eqn (5)), they include the right hand side of the momentum equations (1). Also, there is a missing boundary condition at the left, upstream boundary.*

Eq.(2) - (4) are the momentum equations, in which the boundary conditions are included. In particular, the boundary conditions at free surface:  $\sigma_{ik} n_k = 0$  are included into Eq. (1) and Eq. (2) result from appropriate transformations of basic (i.e. momentum) equations. Similarly, Eq.

(3) result from inclusions of ice shelf base boundary conditions  $\sigma_{ik}n_k = -Pn_i$ , that were written as

$$\begin{cases} \sigma_{xz} = \sigma_{xx} \frac{dh_b}{dx} + \rho g H \frac{dh_b}{dx} + P' \frac{dh_b}{dx}, \\ \sigma_{zz} = \sigma_{xz} \frac{dh_b}{dx} - \rho g H - P', \end{cases} \quad (A1)$$

into the momentum equations.

The approach provides additional stabilizing of the numerical solution. In principle, a finite-difference model, in which the inclusion is performed at ice shelf base only and others boundary conditions are applied in their basic forms (i.e.  $\sigma_{ik}n_k = 0$ ), can be used to obtain ice shelf deflections, but the deflection amplitude exceeds the forcing amplitude too.

Boundary conditions at the left, upstream boundary were considered as the boundary conditions at the summit of symmetrical steady ice sheet, i.e. they are  $U = 0$ ,  $\sigma_{xz} = 0$ . Numerical experiments with simple zero boundary conditions  $U = 0$ ,  $W = 0$  at the left boundary also were carried out to be sure that the effect of the boundary conditions at the left boundary to the deflections is small.

3) *From physical aspects of the presented model, it is unclear why the acceleration terms are necessary. For instance, Sergienko (2010) argues that they could be disregarded due to large speed of the sound waves in ice. The author indicates that the response frequency is the same as the forcing frequency in section 3.2, however, does not make a connection to the (un)importance of the acceleration terms.*

Indeed, in case of a small frequency acceleration term can be neglected, and this is right thing at the first glance. On the other hand, the statement of the eigen-values problem requires to keep these terms and some examples, in which first eigen frequency is a small value, can be given. Thus, if acceleration terms will be disregarded, we can't derive the first eigen frequency. I can give the example from the field of acoustic waves. The spherical resonator (see, for example, Landay, Lifshits, 1987) has the small first eigen frequency, which is defined from the

corresponding wave equation and expressed as  $\omega_0 = c \sqrt{\frac{S}{lV}}$ ; where  $c$  - speed of the sound

waves;  $S$  - square of the cross section of the tube, which is connected to the resonator;  $l$  - length of this tube;  $V$  - volume of the resonator. The design of the resonator suggests that  $S \ll L^2$ , where  $L$  is the size of the resonator. Thus,  $\omega_0 \ll c/L$ , and if we consider the air flow in the tube of the resonator in terms of quasi- steady flow (at the frequency about  $\omega_0$ ) but not as a wave flow, then we will not obtain the first eigen frequency  $\omega_0$ .

Doubtless, the acceleration terms can be disregarded in the case of tidal waves impact to ice shelf.

4) *Figure 10 that shows eigen frequencies and corresponding shelf deflections casts doubts on correctness of the model due to unrealistically large deflection amplitudes. Most likely, there are errors in the model and/or bugs in the code.*

Unrealistically large deflection amplitudes in Fig.10 result from the periodical volume forcing (with non-small amplitude  $\rho g$  per unit volume), which was kept in the eigen frequency problem. From the point of view of ice shelf deflections investigation a periodical volume

forcing should be excluded from the equations, although a number of eigen frequencies, which corresponds to the volume forcing in the glacier, will be missed. The deflection amplitudes in the eigen frequency problem, in which the periodical forcing at ice shelf base only is considered, are in agreement with the values shown in Fig. 7. I agree that the investigated problem implies the exclusion of a periodical volume forcing inside the glacier.

I especially grateful to the referee #2 for detailed revision of the manuscript but the question about the disagreement in the results obtained by the full model and by the model of Holdsworth & Glynn is still opened and is important.

## **References**

- 1) L.D. Landau, E.M. Lifshitz: Fluid Mechanics. Vol. 6 (2nd ed.). Butterworth-Heinemann. 1987
- 2) Holdsworth, G. and J. Glynn: Iceberg calving from floating glaciers by a vibrating mechanism, Nature, 274, 464-466, 1978.
- 3) Vaughan, D. G.: Tidal flexure at ice shelf margins, J. Geophys. Res., 100(B4), 6213-6224, 1995