Interactive comment on “Creep deformation and buttressing capacity of damaged ice shelves: theory and application to Larsen C ice shelf” by C. P. Borstad et al.

J. Bassis
jbasis@umich.edu

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Borstad et al., present an interesting application of damage mechanics to ice shelves. Damage mechanics augments the usual continuum equations with an additional variable that accounts for damage in some homogenized sense. The development of damage mechanics usually proceeds by introducing either a strain equivalence principal or an energy equivalence principal. As the authors note, the strain equivalence principal can be expressed in the form:

\[
\tilde{\sigma} = \sigma \frac{1}{1 - D}
\]

where \(\sigma\) denotes the Cauchy stress and the tilde is used to mark the effective stress.

This relationship applies to fluid parcels that are large compared to the grain size of ice, but small compared to the size of the ice shelf.

The model used to describe ice shelf dynamics in this study relies on an asymptotic, long wavelength depth integrated approximation for deviatoric stress. For example, prior to the application of damage mechanics, the stress equilibrium equations contain integrals of the form:

\[
\int_b^s \sigma_{xx} \, dz
\]

where \(\sigma_{xx}\) denotes one component of the horizontal stress and \(b\) and \(s\) represent the upper and lower surface of the ice shelf, respectively. The stress balance is valid when averaged over length scales that are large compared to the ice thickness. Moreover, the stress depends on depth \(z\) through the depth varying temperature and its influence on the stiffness of ice.

The problem that I’m having is that when one applies the strain equivalence principal to a depth integrated formulation of ice shelf dynamics, the damage \(D\) will depend on the vertical coordinate \(z\). Hence, denoting average quantities with overbars, I would expect that except in special circumstances:

\[
\int_b^s \frac{\sigma_{xx}}{1 - D} \, dz \neq \frac{\bar{\sigma}_{xx}}{1 - D}
\]

Instead, I would expect that one would have to account for the depth variation of damage in computing this integral and sadly, there is not an obvious asymptotic expression for damage that tells us that it should be constant with depth. The problem is that although ice shelf dynamics may be two-dimensional, damage may remain 3D. If the authors have discovered a principal that shows that damage is approximately independent of depth, then this would be outstanding and of great use and it should be elaborated upon in the manuscript.
I think it is very likely that one can use one of the variations of the mean value theorem of Calculus to show that there is some value of \( D \), say \( \hat{D} \), that can be stuffed into the depth integrated expression for stress such that the integrals work out. This \( \hat{D} \) will not necessarily correspond to the strain equivalence definition of damage or interpretation of damage in the usual physical sense, but could still be useful. However, it is difficult to specify an evolution law for the usual definition of damage and there are no evolution laws (yet) for \( D \) that have been proposed. This is especially true if one seeks to incorporate the effect of surface melt water driven hydrofracturing into a damage mechanics law where damage will initiate from the surface and propagate downwards. Including water pressure on the walls is difficult to do even in primitive variable damage mechanics models, much less depth integrated evolution laws.

None of this is meant to be serious criticism of the work discussed here, which is generally of high quality. However, I believe it is important that we are on the same page about the assumptions and approximations that are being considered in this model so that readers not as familiar with the limitations of the damage mechanics theory as the authors clearly are will not be misled or confused.

Interactive comment on The Cryosphere Discuss., 7, 3567, 2013.