Interactive Comment from S. A. Khan (abbas@space.dtu.dk) received and published 13 May 2014

This is a very interesting and well written paper. Hopefully it will be published in its final version soon. However, I have a minor comment regarding your correction on elevation changes.

Page 2344 line 8-10: "Elevation changes were corrected to remove the effect of vertical crustal motion due to Glacial Isostatic Adjustment (GIA) and variations of firn compaction rates in 2003-2009."

Please note that the correction for elastic vertical crustal motion (due to present-day ice loss) is in general much larger than GIA. The elastic rates are typically few cm/yr, while GIA rates are few mm/yr (e.g. see Bevis et al. 2012; Khan et al. 2010a).

To solve the problem, I have uploaded two data files for northeast Greenland. They simply contain rates of elastic vertical crustal motion during 2003-2006 and 2006-2009. Rates are given in mm/yr on a 5x5 km grid and computed as described by Khan et al. 2010b. Feel free to use the data without any restrictions.

To access data use the following link: ftp://ftp.space.dtu.dk/pub/abbas/TCD/

Figure 1:

We thank Dr. Khan for his positive remarks about our manuscript and for providing estimates of the elastic vertical crustal motion during 2003-2006 and
2006-2009. We agree that it is important to remove the effect of this signal from the altimetry derived elevation change estimates. However, the elastic crustal response changes very rapidly in time. Therefore, we believe that the current temporal resolution (3-year average) is not sufficient for correcting our detailed altimetry record. Hopefully, more detailed time series of elastic vertical crustal motion estimates, together with rigorous error estimates, will become available soon, allowing us to completely remove the effect of crustal deformation.

2 Review from A. Aschwanden received on 12 Jul 2014

2.1 General Comment

Initialization remains a serious challenge in ice sheet modeling, and Larour et al. address this challenge by presenting the first use of time-dependent surface altimetry data from ICESat as part of the data assimilation process. Indeed, I’m not aware of any other study to do time-dependent data assimilation. The paper is very well written and structured, it’s easy to read and understand. Adding the temporal dimension to data assimilation opens exiting new capabilities along with many new questions. Therefore I don’t expect the manuscript to answer more questions than the topic raises; and the authors discuss in detail what should be done next. I’m looking forward to read any follow-up papers. The study is certainly worth publishing, and I only have a few comments that I wish to be addressed.

We thank Dr. Aschwanden for his positive review of the manuscript and our approach to assimilating altimetry signals into ice-sheet models. We have taken into account all the detailed comments below, resulting in an improved manuscript. We do agree that many questions are raised by this new type of modeling, and also look forward to working on further studies relying on automatic differentiation and assimilation of altimetry.

For diagnostic (non-transient) case, using surface elevation to constrain the initialization is similar to, but more sophisticated and principled than, flux correction methods used by, e.g. [Price et al. 2011] and [Aschwanden et al. 2013]. A study worth mentioning is [Habermann et al. 2013] as they present snapshots of the evolution of basal yield stress at Jakobshavn Isbrae by inverting surface velocities for a number of years, and find that the observed speed-up is possibly linked to a drop in yield stress. This could be considered as a pre-stage to transient assimilation.

We thank the reviewer for pointing out these references. We have modified the introduction accordingly to include such references.
I’m not an expert in inverse methods myself so I’m not able to judge any technical aspects of the methods presented in the manuscript. It appears all sound to me, but maybe another reviewer could provide more insight.

Dr. Heimbach does indeed provide more feedback on the inversion methodology, which we address in the third review later on.

2.2 Technical Comments

- Equations need proper punctuation (mostly commas are missing after an equation).
  
  We are not sure how to properly handle this comment, as we do not believe that there is missing punctuation in our equations. We will wait for the editor input on this, may be it is TC policy to add punctuation in the equations?

- P. 2335, L. 14-22: Change "in the first section" to "in the next section", and adjust the remainder of the paragraph accordingly.
  Done.

- P. 2340, L. 24 You’ve already used \( n \) for the exponent of the flow law. It’s clear from the context, but you may want to use a different variable.
  Done. Replaced the variable by \( m \). Thank you for catching this.

- P. 2344, L. 20 Maybe I’m missing the obvious, but how can a firn compaction rate be negative?
  
  Negative rates correspond to cases where pore-space increases relatively, by addition of fresh new snow.

- P. 2347, L. 20-23 I’m not sure I understand what you mean with "adjust the overall mean of the entire time series so as to center it...". Could you clarify and add a sentence on how this influences the results?
  
  We refer to a very similar comment by Dr. Heimbach on this subject.

- P. 2348, L. 26. "Matches" sounds very strong and assumes that both variabilities are exactly the same. Are they?
  
  Indeed this statement was too strong. We have reworked this sentence accordingly.

- P.2350, L. 12. "significantly well" is awkward.
  
  Took out the "significantly".

- P.2351, L. 17. "iteration on iteration" is awkward.
  
  Replaced with "after each iteration".

- P.2352, L. 6. "check units, it should be kg.m\(^{-3}\)."
  
  Thank you for catching this typo. Corrected accordingly.
3 Review from P. Heimbach received on 21 Jul 2014

3.1 Main Comments

1. The first general comment is with regard to the choice of control variables relative to the model solved. Eqn. (1), (2) suggest that at every time step, a steady state momentum balance is being solved. The time-dependence enters exclusively through the continuity equation, expressed here as mass/volume conservation equation. This is common practice in ice sheet modeling, but the implication for formulating the control problem should be exposed:

Allowing for a time-varying alpha amounts to adding a time-varying source term in eqns (1), (2), but which are assumed to be steady-state equations. The authors should discuss the interpretation or implications of their approach. It seems to me that the model may be problematic in representing the impact of a time-varying alpha on a time-varying stress balance. This may explain why the optimization of J using the gradient w.r.t. alpha is of limited success. I don’t expect the authors to make changes to their simulations, but to address this issue in the model formulation and in the discussion.

Related to this, I assume that the rationale for making alpha time-varying is that it might be physically connected to time-varying basal lubrication, e.g. through basal melt water (either via seasonal surface melt or geothermal flux or shear heating). I suspect that the main source of time-variability is the expectation that seasonal melt water at the bed would lead to intermittent (early-season) decrease in friction. This is supported
by the discussion on p. 2353 (l. 2-14) of the relationship between basal hydrology and basal stress. However, this is not borne out by the inversion (see Fig. 6a,d). The question then is, what is the physical explanation for time-varying alpha? Alternatively, is the steady-state stress balance appropriate when using time-varying alpha?

Still related, I agree with the interpretation on p. 2349 (l. 14-19) of a "clear equivalence between SMB and surface thickening rate, while basal friction is a direct forcing to the stress-balance equations (Eq. 1) and (Eq. 2), which have no direct bearing on the surface thickening rate", but think this statement needs to be stronger (to re-iterate): The time-varying nature of alpha introduces a time-varying term in steady-state eqns (1), (2), a small inconsistency which the optimization may not be able to handle consistently.

We understand the reviewer’s concern regarding whether our inversion can readily handle the time variable nature of the basal shear at the ice/bedrock interface, and we believe that one of the main goals of our manuscript is to actually demonstrate that time varying basal friction can be inferred accurately from time varying surface altimetry. However, we do not agree that equations (1) and (2) are steady-state equations onto which a time varying basal friction term has been added. Both equations originate from the full stress-balance equation, including acceleration terms which are negligible (which however does not make this a steady-state equation) and time variable stresses. Equations (1) and (2) derive from the stress-balance collapsed using assumptions (1) to (4) (described at lines 2336:14-19) where the time variable term due to friction ($\tau_{bx}$ and $\tau_{by}$) comes from the vertical integration of the $\partial \sigma_{xz} / \partial z$ term in the stress balance. We therefore do not agree with the statement underlying the premise of comment #1 that "allowing for a time-varying alpha amounts to adding a time-varying source term in eqns (1) and (2)". This source term was not added, it is intrinsically part of the stress balance equation, and appears so when formulating the stress-balance according to the SSA formulation. This in our opinion does not explain why the optimization of $J$ using the gradient w.r.t. to alpha is of limited success. Rather, we stand by our assessment in the discussion that the limited success is due to the inherent numerical bias of the mass-transport equations towards SMB and the bias of the stress-balance equations towards surface velocity and friction.

The rationale for making alpha time-varying is indeed that it might be physically related to time-varying lubrication and water pressure. However, it was not our intention to clearly demonstrate a link between seasonality of water runoff (which can transfer into seasonality of the water pressure at the base) and some type of seasonal variability in the evolution of the basal friction. We therefore do not agree with the reviewer’s statement that absence of seasonality in the inferred temporal friction is proof
that our approach is not numerically sound. It rather demonstrates that
surface altimetry is a complex metric which is the result of many complex
processes that cannot be easily inverted for.

Once again, we are not trying to avoid the difficulty in trying to demon-
strate why our basal friction inversion would not be as efficient as the
SMB one. We believe, as the reviewer states, that there is a clear bias in
the inversion where surface altimetry will essentially translate into physi-
cally realistic inversions of SMB, and surface velocities will translate into
physically realistic inversions of friction. We believe we have explained
this quite thoroughly in the discussion.

2. I caution the authors to refer to ”improved” surface heights (e.g. caption
to Fig. 6), or ”improved” M_s. Fig. 6b suggests that S is not im-
proved throughout. Whether all changes in alpha, M_s lead to ”improved”
values is not clear. A better term would be ”adjusted”, i.e. the optimal
values of alpha, M_s are adjusted such as to yield a minimum least-squares
misfit function J. In some cases the adjustment will indeed be improved es-
timates, in other cases, they will compensate for other model or estimation
errors.

We agree with the reviewer’s statements, in particular the fact that a
better fit could indeed be the result of model and/or estimation errors
being compensated for by the parameter being inverted for. We have
modified the text accordingly to remove references to ”improved” surface
heights, in favor of ”improved” best-fit to surface heights, or ”adjusted”
surface heights to observations.

3. p. 2342: The following statement: ”... showing a computation time for
the gradient of the cost function with respect to either alpha or M_s on
the order of 4 times the computation time for the forward model.” simply
cannot be true, unless some very significant shortcuts have been taken.
It is contrary to all accepted wisdom of algorithmic differentiation using
operator overloading versus source-to-source transformation approaches
for complex models. Please either revise this statement, or provide a
description of which shortcuts have been taken, or provide a model setup
that enables testing of this statement by outsiders. (Even if that factor
should turn out to be much larger than 4 times, the author’s achievement
is still very significant).

We do agree with the statement that our factor of 4 cannot be true, if
we assume that the model is completely scalable. Herein lies the critical
difference: we are here dealing with an SSA formulation that relies on the
MUMPS direct solver, which accounts for 90% or more of the computation
time [Larour et al., 2012]. Therefore, during the AD phase of the compu-
tation, the solver still accounts for the majority of the computation time,
which makes our ratio tend towards lower numbers than accepted wisdom
as stated by the reviewer. We now allude to this in the manuscript, so
that AD specialists are not confused by our statement.
4. p. 2342: l. 17/18: The sentence: "the fact that we do not rely on the adjoint-state but rather on AD to compute the gradient, and that the inversion is temporal in nature." is unnecessary and wrong (or a misconception of what AD does). The code generated via AD *does* compute the adjoint state at each time step (no matter which form of AD is used). Therefore, you *do* (have to) rely on computing the transient adjoint state. The only thing you have avoided is having to hand code the adjoint model of your time-varying model that computes this state. AD is only a shortcut for avoiding hand-coding the adjoint model, not a shortcut for avoiding computing the adjoint state.

We do completely agree with the reviewer on this aspect, and have modified the manuscript accordingly.

5. p. 2345: l. 18: The sentence: "Assimilating altimetry data into a forward transient ice flow model presupposes that the model itself is spun-up in a way that more or less closely matches observations for the time period considered." is misleading or wrong. Nothing prevents an assimilation problem to be formulated in such a way that initial conditions and model parameters are adjusted such as to correct a poorly spun-up initial state (e.g., Goldberg and Heimbach, 2013). In fact, "data assimilation" in its most common usage in numerical weather prediction (NWP) is synonymous with finding initial states which lead to optimum fit to observations at analysis time (and optimum forecasts).

A more accurate statement might be: "Since our assimilation method does not adjust initial conditions of the model, we have to rely on a spun-up model state which more or less closely matches observations for the time period considered. In general, the success of inverse methods applied to nonlinear problems often relies, in practice, on initial guesses of the independent variables that yield states that are not too far from observations."

We totally agree with the reviewer, and have actually used the more accurate statement offered.

6. p. 2347: l. 20/21: "Because the model spin-up does not reach a configuration that matches the altimetry time series within a 1 standard deviation, we are still forced to adjust the overall mean of the entire altimetry time series so as to center it on the modeled surface height in 2006." I'm not sure I understand what this means. I think what is being said is that a time-mean bias (spatially constant or spatially varying?) is removed such as to obtain a better initial misfit? This needs to be described more clearly so it is more transparent to readers what is being done. Ideally, a figure should be added, depicting the true mismatch without the adjustment.

This concern has been relayed by the first reviewer also, and we agree that a better explanation is needed. We have reworked the paragraph to show that a spatially varying time-mean has been removed from the altimetry time series, with a figure to more clearly depict it.
7. p. 2347: end of section 3.2: A description is needed regarding the exact nature of the time-variation of alpha and $M_s$. Is the period between two consecutive adjustments the same as the model time step (i.e. two weeks), or is it longer-period? This has repercussion on the dimensionality of the control vector. If $Nx*Ny$ is the dimension of a 2-D field, then the control space would have dimensionality $Nx*Ny*nUpdates$. $nUpdates$ could be either the number of time steps (roughly $[2009-2003+1]*365/14$), or a coarser partition of the integration period. Another question is why the inversion for alpha and $M_s$ have been performed separately (l. 19,20). A formal inversion would invert for both parameters jointly.

We follow the advice of the reviewer and improve the description regarding the nature of the time-variation of alpha and $M_s$. The period between two consecutive adjustments is the same as the model time step (2 weeks). The number of updates we rely on here are therefore the number of time steps in the model. In terms of carrying out the inversion for alpha and $M_s$ separately, we do not yet have the framework required to do multi-parameter inversion, and do not believe we will have it in the foreseeable future, so it is intrinsically a framework restriction. Indeed, multi-parameter inversions would be more effective, and this will be hopefully carried out in further studies.

8. p. 2348: l. 2/3: I am not sure how it can be inferred from Fig. 4 that "best-fit to observations can only be improved by varying forcing over the entire space and time domain." All that Fig. 4 shows is that the gradients are space-time dependent. This, in turn, is a consequence of the nature of the observations. To see this, note that for a cost function of form:

$$J = \frac{1}{2} (F(x) - obs)^2,$$

the gradient is of the general form:

$$dJ/dx = (dF/dx)^T * (F(x) - obs),$$

i.e. the gradient is "driven" by the (linear) model $(F(x))$ vs. data $(obs)$ misfit. To the extent that $(F(x) - obs)$ is time-space varying, so will be the gradient.

We agree with Dr. Heimbach and thank him for the explanation of how the gradient variability is intrinsic to the nature of the cost function we chose for the given temporal inversion. We improved the paragraph accordingly by showing how the results indeed convey the expected variability, and replicated the equation demonstrating the time-space variability of the gradient for a quadratic cost function.

Related, p. 2348, l. 12-15 and p. 2349, l. 5-7: "For $dJ/dalpha$, this can be largely explained by the fact that basal friction is much higher there than near the coastline, making it much harder for equivalent variations in basal friction to impact ice flow dynamics and surface heights."

This may be the case, but is an interpretation not readily borne out by
the analysis. The simplest explanation that is supported by the analysis is the same as above, i.e. the fact that:

\[ \frac{dJ}{dx} = \frac{dF}{dx} \times (F(x) - \text{obs}) \]

implies that for small misfits \((F(x) - \text{obs})\), which is the case inland, the gradient is small, no matter what the size of \(x\) (here = alpha), unless \((dF/dx)^T\) itself would be very large (but which too would require demonstration). Linking the smallness of \(dJ/d\alpha\) to the largeness of alpha itself requires further scaling analysis.

We agree with the reviewer, and again thank him for the clear explanation of why gradients inland are expected to be smaller. As Fig 8a and 8d indeed shows, the misfit pre-inversion is already small inland, which implies that this will also be the case for the gradient. We therefore agree that linking the smallness of \(dJ/d\alpha\) to the largeness of alpha itself requires further scaling analysis. We refined our statement in p.2348, l. 12-15 accordingly.

Still related, the "controlling mechanism" invoked on p. 2349 (l. 5-7) can instead be simply explained by the small residual model-data misfit in the regions suggested upward of the suggested demarcation.

We agree that the "controlling mechanism" is more simply explained by the low initial misfit. We modify the text accordingly.

9. p. 2350, l.9 onward: Figures 6c, g suggest that the optimization "corrects" winter mass balances for both positions I, II to be solidly negative, compared to their first-guess values which are near zero or slightly positive. Is this expected? The implication would of a negative mass balance not just during summer months but throughout the year would seem significant. This is indeed not expected, and is due to the fact that we have not constrained the input \(M_s\) to be within a certain error margin of its original value, hence leading to winters where there can be melt-rates, and summers that freeze. This is an issue in the current implementation of the method, which will be corrected in further studies. We modified this paragraph to make sure that the implication that there is a negative mass balance throughout the year is not inferred from our study.

3.2 Details

- p. 2332 l. 20: It seems more prudent to refer to the common terminology Global Mean Sea Level (GMSL) rise. Alternatively, refer to "sea level change", since regional sea level trends may be negative (i.e. sea level drop) over the last 20 years.

Agreed, we now use Global Mean Sea Level instead of Sea Level.

- p. 2332 l. 21: Update to IPCC AR5 (plus relevant reference)
Done. We now refer to the Physical Science Basis [Stocker et al., 2013].

• p. 2335: l. 14-21: correct all section numbers (section N -¿ section N+1)
  Done. Thank you for spotting this issue.

• p. 2335 l. 15: Here and throughout the manuscript (e.g., p. 2336, l.5; etc.) it would seem "nicer" and consistent with the estimation/control theory literature, to refer to "objective function" instead of "diagnostic".
  Agreed. We have referred to "objective function" throughout the manuscript in accordance with the estimation/control theory literature.

• p. 2336: l. 8: Replace "Ice flow on the NEGIS" -¿ "Flow of the NEGIS" (seems to me that the ice doesn't flow *on* the NEGIS, and "Ice flow" of the "Ice Stream" seems redundant).
  Done.

• p. 2339: l. 2: The cost function sums the SQUARED differences.
  Indeed. Thanks for spotting the issue. Corrected.

• p. 2339 l. 16: Here, and later in the manuscript (e.g., p. 2342) the notation \( J = F(\alpha(t),M_s(t)) \) is not well defined, or misleading. If \( F \) indeed refers to the model (defined how? I guess the system of eqns. (1) to (6)) then \( J \) is not scalar-valued. Instead, I think what you mean is: \( J = J(F(\alpha(t),M_s(t))) \)
  Indeed this is what was meant. We corrected accordingly.

• p. 2340: l. 8: Replace "adjoint theory" by "adjoint method"
  Done.

• p. 2342: l. 8: Reword: "... we can AD-compute \( dJ/d\alpha \), gradient of..." to "... we can compute \( dJ/d\alpha \), the gradient of..."
  Done.

• p. 2342: l. 12: It might be more conceptually more transparent to distinguish between first- guess \( \alpha_0 \) and optimized \( \alpha = \alpha_0 + \Delta \alpha \), i.e. write: "... we can infer an update \( \Delta \alpha \) to \( \alpha_0 \), such that \( \alpha = \alpha_0 + \Delta \alpha \) leads to a simulated surface height evolution that minimizes the cost function".
  Thank you for the suggestion, which we integrated as is into the text.
• p. 2342: l. 13/14: Not the "inverted" alpha itself best fits the data, but the state computed with the adjusted alpha does.

Done.

• p. 2342: l. 22, 24/25: "Here, we do not assimilate both forcing alpha and $M_s$." This statement is wrong, it mixes up dependent and independent variables. Observations are assimilated, not input variables. What you mean is either: "we do not invert for both ..." or "we do not adjust both ...". Likewise, the sentence: "which parameter assimilates existing altimetry observations most efficiently" is ill-worded.

Thank you for catching this inconsistency. We amended the manuscript accordingly.

• p. 2343: l. 6: "by a simultaneous reconstruction of the surface topography" Use of *simultaneous* makes sense for reconstruction of A *and* B. A is surface topography. What is B? Otherwise drop *simultaneous.

Indeed. We dropped simultaneous.

• p. 2346 l. 1/2: "instantaneous spin-ups" This seems a bit of an oxymoron (or the term "spin-ups" misleading), so perhaps add "or snapshot inversions". Also, in the following reference list, it seems warranted to add Petra et al. [2012].

Indeed it is an oxymoron, but I don't believe the terminology "snapshot inversion", though more accurate, is used in the Glaciological Community. I would ask that we do not change this term. We added Petra et al. [2012] to the list of references.

• p. 2346 l. 6-9: It may be true that: "However, this approach relies on a steady-state thermal regime for the ice sheet, which is not realistic, ..." but the same is true for the approach presented here, see p. 2888, l. 3-5: "The thermal regime of the ice is not captured in our transient ice flow model ... We believe this approximation to be realistic". It would seem that the approximation made holds equally well in both cases. By the same token, the statement "usually leads to lumping any mismatch between model and observations into the inversion itself" is equally valid in both cases, to the extent that it refers to the thermal regime.

Indeed, the approach presented here also relies on a steady-state thermal regime, we therefore cannot use this argument. Same holds for the error lumping argument. The difference actually lies in the fact that the instantaneous spin-up relies on a steady-state also for the mechanical stress-balance. We added "mechanical" to "thermal steady-state" to explain
our rationale.

• p. 2346 l. 21: "followed by a relaxation of the ice sheet/ice shelf over a period of 50,000 years" I am not sure what this means, or whether this is a common numerical method. I suggest describing what the "relaxation" involves (in fact, some authors refer to "relaxation" as a simple form of data assimilation, but I suspect this is not implied here?).

By relaxation, we mean a transient run where all the forcing are kept constant. We added this description in the paragraph.

• p. 2346 l. 22: "The climate forcing is constrained by an SMB taken equal to ...". I don’t understand what is meant here by "constrained". I suspect the authors simply mean: "The climate forcing is represented by the time-mean SMB between 1971 and 1988". Similarly, it is somewhat unclear to me why the period 1971-1988 is chosen as "climatology". The Box et al. (2013) time series goes back to 1840, so why not taking 1850-1988 as a more representative climatology (i.e. a better average over decadal variability), or any other start date between 1840 and 1971? If the 1971-1988 time-mean is used for the integration prior to 1971, it would seem more likely that SMB undergoes an artificial jump in 1840 (the time at which the Box et al. time series is applied) than using a time-mean SMB which is more representative to 1840 (?)

We understand the rationale of the reviewer here, but we believe that the 1971-1988 is more representative of a steady-state ice sheet that would be better suited for a relaxation algorithm. There will indeed be an artificial jump when we connect the relaxation over 50,000 years to the LIA-present period, but this will avoid having to trust data that is less reliable the further back we go in the Box et al. [2013] time series. We added some description to this effect.

• p. 2348, l. 28 / p. 2349, l.1/2: I have difficulties seeing the "clear demarcation line" and "abrupt transition in ice thickness". I’d suggest adding corresponding isolines/ contours to Figs. 4 and 7 that delineate the transitions in question.

We color coded the 1000 m contour level in Fig. 4 (now Fig. 5) to help delineate the transition. We now refer to the contour in the manuscript.

• p. 2349, l. 19/20: Reword "... between both methods ..." to "... between varying alpha or $M_s$ ..."

Done.

• p. 2353, l. 1: "... exhibits high variability ..." In space or time, or both?

Both indeed. Done.
• p. 2353, l. 28/29 and p. 2354, l. 3-6: "Here, we propose..." A good proposition, one that has already been formulated by Heimbach and Bugnion (probably others before), and that has already been explored by Goldberg and Heimbach (2013), who used time-varying altimetry and surface velocities with inhomogeneous temporal sampling (to reflect heterogeneous InSAR vs. ICESat sampling, albeit in a synthetic experiment) to constrain a transient ice flow model and simultaneously infer best-estimate initial conditions and basal sliding.

Apologies for repeating some idea already published, we now make reference to both suggested references in the manuscript.
References


Inferred basal friction and surface mass balance of North-East North East Greenland Ice Stream using data assimilation of ICESat-1 ICESat surface altimetry and ISSM

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Abstract

We present a new data assimilation method within the ISSM framework that is capable of assimilating surface altimetry data from missions such as ICESat-1 into reconstructions of transient ice flow. The new method relies on algorithmic differentiation to compute gradients of diagnostics with respect to model forcings. It is applied to the North East Greenland Ice Stream where surface mass balance and basal friction forcings are temporally inverted, resulting in significantly improved adjusted modeled surface heights that match best-fit existing altimetry. This new approach allows for a better quantification of basal and surface processes, and a better understanding of the physical processes currently missing in transient ice flow models to better capture the important intra and inter-annual variability in surface altimetry. It also demonstrates that large spatial and temporal variability is required in model forcings such as surface mass balance and basal friction, variability that can only be explained by including more complex processes such as snowpack compaction at the surface and basal hydrology at the bottom of the ice sheet. This approach is indeed a first step towards assimilating the wealth of high spatial resolution altimetry data available from EnviSat, ICESat-1, Operation IceBridge and CryoSat-2, and that will be available in the near future with the launch of ICESat-2.

1 Introduction

Current sea-level rise (SLR) - Global mean sea level (GMSL) rise observations show an overall budget in which fresh-water contribution from the polar ice sheets represents a significant portion (IPCC-AR4, 2007; Church and White, 2006, 2011; Church and White, 2006, 2011; Stocker et al., 2013), which is actually increasing (Velicogna, 2009; Rignot, 2008) relative to thermo-steric expansion and contribution from terrestrial glaciers (Gardner et al., 2013). In order to quantify the contribution of polar ice sheets to SLR - GMSL in the near future, accurate mass balance projections must therefore be carried out, which can either be based on extrapolation of current trends (Velicogna and Wahr, 2006; Velicogna, 2009; Shepherd and Wingham, 2007; Rignot,
et al., 2011), or supported by transient ice flow models that are physically validated against data. Such models, however, as demonstrated in the SeaRISE (Sea-level Response to Ice Sheet Evolution) and ice2sea inter-comparison projects, are not fully capable of capturing the present-day trends (Bindschadler et al., 2013; Nowicki et al., 2013a,b), which hinders our ability to project them into the future with a high degree of confidence. Indeed, one of the critical difficulties faced by ice-sheet modelers is the spin-up of ice-flow dynamics in a way that matches present-day observations. At the root of the problem is a complicated interaction between: 1) paleo-reconstructions that can match the evolution of the ice volume for the Greenland Ice Sheet (GIS) or Antarctic Ice Sheet (AIS) (Pollard and DeConto, 2009; Huybrechts et al., 2011; Ritz et al., 1997; Greve, 1997a), but which fail at capturing present-day ice-flow dynamics and 2) what is now referred to as "instantaneous spin-ups" that rely on inversion of basal friction at the ice/bed interface from satellite-derived surface velocities (MacAyeal, 1993; Morlighem et al., 2010; Seroussi et al., 2013; Price et al., 2011; Arthern and Gudmundsson, 2010), which are more efficient at capturing present-day ice-flow dynamics, but lack the long-term trends in the stress and thermal regime of both ice sheets.

In order to bridge the gap between both approaches, so that paleo reconstructions of ice sheets and present-day dynamic conditions can be captured in one continuous run of an ice sheet model, there needs to be a shift in the way we approach integrating data into transient ice flow models. Indeed, there is a wealth of data available to ice sheet modelers that is not yet fully leveraged to constrain transient ice flow models of the GIS and AIS. These data include among others: 1) Ice coring that provide temperature profiles for calibration of thermal models (Greve, 1997b, 2005); 2) Radar stratigraphy that provide layering and age structure (Fahnestock et al., 2001); 3) Sediment coring in pro-glacial lakes around the GIS (Briner et al., 2010, 2011) that provide vital information about margin positions throughout the Holocene; 4) High spatial resolution surface altimetry from small footprint satellite laser altimeters onboard missions such as EnviSat (2002-2012), ICESat (Ice Cloud and land Elevation Satellite, 2003-2009) and CryoSat-2 (2010-present) as well as ESA and NASA airborne laser altimetry campaigns flown as part of PARCA (1993-2008) (Thomas and Investigators, 2001; Thomas et al., 2004) and Operation IceBridge (2009-present) that provide an almost continuous surface height record.
for the past 20 years (Csatho et al., 2013; and 5) SAR (Synthetic Aperture Radar) data from missions such as ERS-1,2 (European Remote Sensing Satellites), RADARSAT-1,2, ALOS (Advanced Land Observing Satellite) PALSAR (Phased Array type L-band Synthetic Aperture Radar), TerraSAR-X among others, that provide a surface velocity record (albeit relatively discontinuous compared to the altimetry record) for the last 20 years (Joughin et al., 2004a, 2010, Mouginot et al., 2014). All these datasets are yet to be systematically assimilated into ice sheet models, to provide better constraints for transient models.

One of the main difficulties in reconstructing past and present ice-sheet flow that is compatible with observations and ice-flow dynamics is the lack of temporally variable assimilation methods. Most attempts at assimilating surface altimetry so far have relied on approaches such as indirect approaches. The first one is ensemble methods, where sub-sets of model runs that are compatible with present-day conditions of the ice sheet are down-selected (Aschwanden et al., 2013). This type of method does improve spin-ups and inform about the level of uncertainty inherent in the model runs, but does not yield information on the underlying boundary conditions, and potential corrections that have to be applied (within specific measurement error margins) for the model to converge to present-day conditions. For example, one such boundary condition that is inherently difficult to reproduce from the previous inter-glacial onwards is surface mass balance (SMB) (Ritz et al., 1997), which is a critical component of the GIS and AIS mass transport. The second method is the so-called flux-correction methods (Aschwanden et al., 2013, Price et al., 2011) where boundary conditions at the ice front are corrected for in order to match time series of observed fluxes. This is a direct tuning approach which however can be incompatible with realistic boundary conditions at the ice front. The third and final method is a quasi-static approach, in which surface velocities is used at different snapshots in time to invert for parameters such as basal yield stress. This approach is used in particular in Habermann et al. (2013) to understand the dynamics of Jakobshavn Isbrae in the past two decades. Such methods are indeed precursors to more sophisticated methods which can tightly couple mass transport and stress balance in time-dependent assimilations.

New approaches are however emerging, based on time-dependent adjoint modeling (Heimbach, 2008, Goldberg and Heimbach, 2013), which show
great promise in potentially enabling sophisticated data assimilation of disparate datasets into ice sheet models. Here, we propose one such approach, based on algorithmic differentiation of the Ice Sheet System Model (ISSM), to improve assimilation of surface altimetry data (over the entire ICESat – ICESat time period) into a transient reconstruction of the ice-flow dynamics of the North-East Greenland Ice Stream (NEGIS). The goal here is to invert for temporal forcings, namely SMB and basal friction at the ice/bed interface, such that best-fit to surface altimetry data is achieved, while respecting the constraints inherent in the physics of an ice-sheet flow model. On NEGIS, altimetry data provided by ICESat – ICESat displays high levels of spatial and temporal variability, which presents a good case scenario for investigating how model forcings need to be corrected for in order to replicate such variability in surface height, and what are potentially missing components in the way we model surface and basal processes that should be improved accordingly. In addition, recent studies suggest that NEGIS is undergoing dynamic thinning linked to regional warming [Khan et al., 2014], which will exhibit characteristic surface altimetry signatures that should be investigated, especially for a basin that previously was considered stable.

This study is structured as follows: in the first next section we describe our forward transient model, the equations, the diagnostics objective function we are interested in, and the algorithmic differentiation approach to computing gradients of such diagnostics objective function along with the temporal inversion algorithm necessary to infer temporally variable model forcings that best-fit observations. In section twothree, we describe the altimetry time-series used on NEGIS, our spin-up methodology and all model inputs to our runs. Sections three and four four and five present our results and discussion respectively, and section five the final section concludes on the implications of our new approach towards assimilating altimetry data into projections of ice-sheet mass balance.

2 Model

Underlying our assimilation of surface altimetry is a transient ice-flow model, which can simulate the temporal evolution of an ice stream. We refer to this model as our forward model. This
model is implemented within the Ice Sheet System Model (Larour et al., 2012c; Morlighem et al., 2010). Here, we are interested in the best-fit between modeled surface heights and available altimetry observations. The gradient of this diagnostic value with respect to transient forcings (basal friction at the ice/bedrock interface and SMB) can then be used within an inverse method to invert for such forcings, while simultaneously improving our best-fit to observations. In this section, we describe our forward model, the computation of the diagnostic objective cost function and the methodology behind the computation of the gradient as well as the inverse method itself.

2.1 Forward model

Ice flow on the NEGIS is characterized by low basal shear stress across the entire basin (Joughin et al., 2001; Schlegel et al., 2013), resulting in high velocities (Fahnestock et al., 2001) deep inland towards the ice divide (cf. Fig 1). Such flow exhibits low vertical shear stress, and can therefore be realistically described using the Shelfy-Stream Approximation (SSA) (MacAyeal, 1989). This formulation is a simplification of the full-Stokes equations describing the stress balance of an ice sheet. The simplifications involved include: 1) neglecting the ice-flow acceleration (Reist, 2005); 2) neglecting horizontal gradients of vertical velocities compared to vertical gradients of horizontal velocities (Blatter, 1995; Pattyn, 2003); 3) neglecting bridging effects (van der Veen and Whillans, 1989) and 4) neglecting vertical shear altogether (which encompasses assumption 2 also) (MacAyeal, 1989).

Using 1) to 4), the stress balance equilibrium can be reduced to the following two equations expressed in terms of the depth-averaged horizontal velocity $(u,v)$:

\[ \frac{\partial}{\partial x} \left( 4h\mu \frac{\partial u}{\partial x} + 2h\mu \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial y} \left( h\mu \frac{\partial u}{\partial y} + h\mu \frac{\partial v}{\partial x} \right) = \rho gh \frac{\partial s}{\partial x} - \tau_{bx} \] (1)

\[ \frac{\partial}{\partial y} \left( 4h\mu \frac{\partial v}{\partial y} + 2h\mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial x} \left( h\mu \frac{\partial u}{\partial y} + h\mu \frac{\partial v}{\partial x} \right) = \rho gh \frac{\partial s}{\partial y} - \tau_{by} \] (2)
where $\mu$ is the depth-averaged ice viscosity, $\rho$ the ice density, $g$ the acceleration due to gravity, $h$ the local ice thickness, $s$ the upper surface elevation and $(\tau_{bx}, \tau_{by})$ the x,y components of the basal shear stress at the ice/bedrock interface. In this formulation, horizontal velocity is considered independent of $z$, and vertical velocity can be recovered through the incompressibility condition. In the case of NEGIS, the simplifying assumptions 1) to 4) are valid for the entirety of the basin, excluding some very specific areas where some departure to the SSA formulation occurs. For more details on these areas, we refer to Schlegel et al. (2013).

The basal shear $\tau_b$, expressed in terms of horizontal components of the basal shear $(\tau_{bx}, \tau_{by})$ in equations (1) and (2) is a model forcing which we describe using a viscous linear relationship (MacAyeal, 1989, 1993):

$$\tau_b = -\alpha^2 N_{eff} v$$  

(3)

where $v$ is the velocity parallel to the ice/bedrock interface, approximated here as $(u,v)$, $\alpha$ the friction coefficient and $N_{eff}$ the effective water pressure taken here equal to the pressure at the ice base.

Viscosity in equations (1) and (2) is described using the following Norton-Hoff law (Glen, 1955):

$$\mu = \frac{B}{2\dot{\varepsilon}_e^n}$$

(4)

where $B$ is the ice hardness, $n$ Glen’s law exponent and $\dot{\varepsilon}_e$ the effective strain rate. $B$ is temperature dependent and follows an Arrhenius type law calibrated from Paterson (1994).

The SSA formulation solves for the stress balance. However, mass conservation also needs to be ensured through the following mass transport equation:

$$\frac{\partial h}{\partial t} + \nabla \cdot h \mathbf{v} = M_s - M_b$$

(5)

where $h$ is the ice thickness, $\mathbf{v} = (u,v)$ the depth-averaged velocity, $M_s$ the surface mass balance (m.a$^{-1}$ of ice equivalent) positive for accumulation, negative for ablation and $M_b$ the basal melting rate (m.a$^{-1}$ in ice equivalent), positive for melting, negative for freezing.
The thermal regime of the ice is not captured in our transient ice flow model. It is initialized using a thermal steady-state model described in Larour et al. (2012b), which is then assumed constant through time. We believe this approximation to be realistic, assuming there are no short-term thermal transients that develop between 2003 and 2009, the length of our data record (Seroussi et al., 2013).

In terms of boundary conditions, (1) and (2) are solved using observed Dirichlet surface velocity constraints at the boundaries inland, stress-free surface, friction at the ice/bedrock interface (described by (3)) and water pressure at the ice front, which when depth-averaged, results in the following condition:

$$\sigma \cdot n = \left( \frac{1}{2} \rho g H^2 - \frac{1}{2} \rho_w g \min(b, 0)^2 \right) n$$

(6)

where $\sigma$ is the stress tensor, $n$ the unit outward-pointing normal vector at the ice front, $\rho_w$ the water density, and $b$ the elevation of the ice lower surface. At the ice front, the boundary condition for mass transport (5) is specified as a free-flux boundary condition, where the calving rate is taken equal to the normal velocity at the ice front.

Equations (1), (2), (4) and (5) along with corresponding boundary conditions, can be discretized and solved implicitly in time using the Finite Element Method (FEM). We refer to Larour et al. (2012c) for more details on the FEM discretization as well as numerical schemes to handle the material non-linearity and the stability of our time-stepping.

2.2 Cost function

Our forward model computes, from model inputs $\alpha$ (friction coefficient) and $M_s$ (surface mass balance), time series of surface heights $s(t)$ and depth-averaged horizontal velocities $(u(t), v(t))$. The diagnostic quantity considered here for our forward model is however not the surface height $s(t)$ nor horizontal velocities $(u(t), v(t))$, but rather the cost function $J$ which describes the time and space-averaged squared difference between the modeled surface heights $s(t)$ and the observed surface elevations from ICESat–ICESat altimetry. If we name $s(t)_{obs}$
the time evolving altimetry observations, we can define our cost function as:

\[
J = \frac{1}{S_\Omega} \frac{1}{T} \int_{\Omega} \int_{t=0}^{t=T} \frac{(s(t) - s(t)_{obs})^2}{2} \, d\Omega \, dt
\]  

(7)

where \( \Omega \) is the spatial domain (here the entire NEGIS basin), \( S_\Omega \) its surface extent, and \([0,T]\) the time domain over which ICESat-1 observations are available. This cost-function describes the time-averaged and space-averaged misfit to altimetry observations. It can be decomposed also into a spatial average of a local time-averaged misfit \( J_T(x,y) \), as follows:

\[
J = \frac{1}{S_\Omega} \int_{\Omega} J_T(x,y) \, d\Omega
\]  

(8)

where \( J_T \) is defined as:

\[
J_T(x,y) = \frac{1}{T} \int_{t=0}^{t=T} \frac{(s(t) - s(t)_{obs})^2}{2} \, dt
\]  

(9)

In the remainder of our study, we assimilate temporally variable frictions \( \alpha(t) \) and surface mass balance \( M_s(t) \) in order to minimize the cost-function \( J=F(\alpha(t),M_s(t)) \) where \( F \) is the forward model described in equations (1), (2), (4) and (5). This means that we temporally invert for the friction and SMB that best-fit our observations. Our initial values for both forcings come from 1) the time variable SMB from Box (2013) and 2) a time-constant friction which is variable in space, inferred using an adjoint-based inversion of existing present-day surface velocities (Morlighem et al., 2010; Rignot and Mouginot, 2012). The main results of our temporal inversion are temporal corrections to these forcings, and an improved best-fit (or minimized \( J \)) to observations.

2.3 Algorithmic Differentiation of the gradient of \( J \)

The basis for inverting forcings \( \alpha \) and \( M_s \) is the determination of time and space dependent gradients of our cost function \( J \), namely \( \partial J/\partial \alpha(x,y,t) \) and \( \partial J/\partial M_s(x,y,t) \). A common approach in the Cryosphere community to obtain gradients of forward models has been to rely...
on the adjoint theory-method (MacAyeal, 1993; Rommelaere and MacAyeal, 1997; Vieli and Payne, 2003; Joughin et al., 2004b; Larour et al., 2005; Vieli et al., 2006; Khazendar et al., 2007, 2009; Morlighem et al., 2010; Arthern and Gudmundsson, 2010). The approach consists in analytically deriving the adjoint state of the forward model, which allows for an easy computation of the gradient of the cost function. This approach works particularly well for self-adjoint models, which is the case for the stress balance equations of ice flow when the rheology is considered linear viscous. When non-linearities are present in the model, as is the case when relying on a material law such as (4) with \( n \neq 1 \), the adjoint-approach can still be viable if the problem is linearized (Morlighem et al., 2013b) or if an exact adjoint can be analytically derived.

For more complex ice-flow models however, the adjoint state is usually not easily derivable, and other methods have to be considered to compute cost-function gradients. The first one is to rely on approximation by forward-differencing, as in Larour et al. (2012b,a); Schlegel et al. (2013). This method is flexible (it can be applied to any type of forward model) but it is computationally expensive. Indeed, for a given cost function and a model input of size \( n \times m \) (\( m \) being the number of degrees of freedom, such as the mesh size for an FEM discretization, or the grid size for a Finite Difference discretization), this method requires at least \( n+1 \times m+1 \) computations of the forward model. For transient ice flow models which are computationally expensive, this is therefore impractical. Furthermore, for highly nonlinear models the choice of perturbation greatly affects the quality of the approximation.

The second method relies on algorithmic differentiation (AD) of the forward model, where the derivative computation is enabled by semantically augmenting the computer program that implements \( F \) (Griewank and Walther, 2008). The AD approach has been implemented for common programming languages in a variety of tools. Such tools include source-to-source transformation frameworks (TAF, Giering et al. (2005); OpenAD, Utke et al. (2008) and Tapedenade, Hascoët (2004)) and/or overloaded operator frameworks (ADOL-C, Griewank et al. (1996); Walter et al. (2012)). AD tools can automatically generate derivatives (first-order gradients, Taylor type developments, Hessians) of the forward model at machine precision and at a computational cost that, unlike the cost of forward-differencing methods, is a small fixed factor
independent of $m$. Typically, source-to-source transformation tools can compute a gradient of the forward model in 4-10 times the cost of the forward run itself. This can be somewhat higher (but still fixed) for overloaded-operator approaches, which are not computationally as efficient. AD tools have been leveraged extensively in the oceanic context for state-of-the-art ocean models (Marotzke et al., 1999; Heimbach et al., 2002; Heimbach, 2008), and more recently to ice-sheet models (Heimbach and Bugnion, 2009; Goldberg and Heimbach, 2013).

In this paper we work with ISSM, which is written in C++ and uses a variety of object-oriented features. Because of the complexity of the C++ syntax and semantics, there currently are no AD tool for the comprehensive source code transformation of C++ models. Thus the operator-overloading approach becomes the method of choice. In this type of approach, the floating-point operations on quantities which are part of the computation to be differentiated are recorded into a tape during the forward run. This tape can then be interpreted in a reverse sweep to compute the gradient of the cost-function with respect to model inputs, using the chain rule in reverse order. Here, the underlying library used to implement the overloading of our floating-point operations in ISSM is Adol-C (Walter et al., 2012). Testing and validation of the modifications were carried out against the forward difference approach implemented in Larour et al. (2012b). Benchmarks for computation times were also carried out, showing a computation time for the gradient of the cost function with respect to either $\alpha$ or $M_s$ on the order of 4 times the computation time for the forward model. This is for models on the order of 2000 degrees of freedom, and which represents a very efficient ratio of performance. Indeed, this ratio is much lower than the currently accepted ratio of 10 expected of operator-overloading approaches. This is explained by the low scalability of the direct solver used in our computations (Larour et al., 2012c) which accounts for 90% and more of the computation time in forward and AD mode, making the ratio appear much lower than for fully scalable solutions.

2.4 Inverse method

If we restrict ourselves to the case of temporally inverting basal friction (the same logic applies to surface mass balance), we can AD-compute $\partial J/\partial \alpha(x,y,t)$, the gradient of our cost function $J$ with respect to basal friction $\alpha(x,y,t)$. The cost function is computed using the
forward model \( J = F(\alpha(x,y,t)) \). \( \alpha \) is variable both in space and time, as well as the gradient itself. Using a classic steepest descent along the vectorial direction set by \( \partial J / \partial \alpha(x,y,t) \), we can infer \( \alpha(x,y,t) \) that an update \( \Delta \alpha \) to the initial \( \alpha_0 \), such that \( \alpha = \alpha_0 + \Delta \alpha \) leads to a simulated surface height evolution that minimizes the cost function \( J \).

Once a minimum \( J \) is reached, we have effectively determined a new ”inverted” \( \alpha \), which best-fits which the modeled ice-sheet height to best-fits surface altimetry. The difference between this algorithm and the classic approach presented in inversion studies such as MacAyeal (1993); Morlighem et al. (2010); Arthern and Gudmundsson (2010) is the fact that we do not rely on need to hand-derive the adjoint-state but rather on AD to compute the gradient of the model, as it is automatically computed by the AD gradient operator, and that the inversion is temporal in nature.

This inversion is also a data assimilation in that we compute corrections that have to be applied to an existing time series of forcings (here basal friction or surface mass balance) in order for a certain diagnostics objective function of our model to match observations. Here, we do not assimilate invert for both forcings \( \alpha \) and \( M_s \) at the same time. Rather, we invert \( \alpha \) given observed surface mass balance \( M_s \), and vice-versa. This approach allows us to better understand which parameter assimilates results in the best assimilation of existing altimetry observations most efficiently, and what type of physical processes are involved in best-fitting the model results to observed data.

### 3 Data and Model Setup

#### 3.1 Altimetry

Elevation changes during the 2003-2009 period were reconstructed from ICESat-1 ICESat laser altimetry observations using the Surface Elevation Reconstruction And Change detection (SERAC) approach (Schenk and Csatho, 2012). This comprehensive method was developed to determine surface changes by a simultaneous reconstruction of the surface topography. Although general in the design, SERAC was specifically developed for detecting changes in ice
sheet elevation from ICESat-1 ICESat crossover areas. It addresses the problem of computing a surface and surface elevation changes from discrete, irregularly distributed samples of the changing surface. In every sampling period, the distribution and density of the acquired laser points is different. The method is based on fitting an analytical function to the laser points of a surface patch, for example a crossover area, size $1\,\text{km} \times 1\,\text{km}$, or within repeat tracks, for estimating the ice sheet surface topography. The assumption that a surface patch can be well approximated by analytical functions, e.g. low order polynomials, is supported by various roughness studies of ice sheets (van der Veen et al., 2009). Considering physical properties of solid surfaces and the rather small size of surface patches suggests that the surface patches are only subject to elevation changes but no significant deformations. That is, the shapes of the 1 by 1 km surface patches centered at the crossovers remain constant and only the vertical position changes as it is confirmed by the low surface fitting errors obtained by SERAC (Schenk and Csatho, 2012). It is important to realize that SERAC determines elevation changes as the difference between surfaces, unlike other methods that take the difference between identical points of two surfaces.

Laser points of all time epochs of a surface patch contribute to the shape parameters while the laser points of each time period determine the absolute elevation of the surface patch of that period. Since there are many more laser points than unknowns, surface elevation and shape parameters are recovered by a least squares adjustment whose target function minimizes the square sum of residuals between the fitted surface and the data points. The large redundancy makes the surface recovery and elevation change detection very accurate and robust. Moreover, the confidence of the results is quantified by rigorous error propagation.

With SERAC we reconstructed time-series of elevation change histories at 837 ICESat-1 ICESat ground-track crossover locations within the NEGIS drainage basin (Fig. 2). Assuming that the laser points entering the adjustment model are uncorrelated and have all the same weight, the random errors of elevation as a function of time is determined from the variance-covariance matrix of the normal equation. Elevation changes were corrected to remove the effect of vertical crustal motion due to Glacial Isostatic Adjustment (GIA) and variations of firn compaction rates in 2003-2009. Indeed, ISSM is a model that relies on the assumption of
incompressible ice flow, and the surface elevation must therefore be converted from a snow/ice equivalent (where density throughout the firn layer is variable) to an ice equivalent (where we assume a through-thickness density profile that is constant and equal to 917 kg/m$^3$). GIA-related vertical crustal motion estimates are from A.G. and Zhong (2013), based on ICE-5G ice history, a VM2 viscosity profile and a 1 by 1 degree mesh. Estimates range from -2.7 mm/yr to 4.6 mm/yr, with errors that are negligible compared with elevation changes due to other factors. Variations of firn compaction rates above the equilibrium line altitude (ELA) are from a 5 km by 5 km gridded model forced by the output from the HIRHAM5 Regional Climate Model (Sørensen et al., 2011; Lucas-Picher et al., 2012) and range from -0.016 m/yr to 0.146 m/yr. Finally the corrected elevation changes were converted into ice equivalent elevation changes using a constant ice density of 917 kg/m$^3$ in the ablation and superimposed ice zones and a simple firn-densification model from (Reeh et al., 2005; Reeh, 2008) in the accumulation zone. This model assumes that all retained melt water (Superimposed Ice Remaining at the end of the melt season (SIR)=melt - runoff) refreezes at the same annual layer at the end of each balance year (August 31), giving

$$\rho_s = \frac{\rho_0}{1 - \frac{SIR}{M_s} \left(1 - \frac{\rho_0}{\rho_{ice}}\right)}$$

(10)

where $\rho_s$ is the density of the annual firn layer on the surface, $SIR$ is the amount of refrozen ice, estimated as the difference between the annual melt and runoff, $M_s$ is the annual net surface mass balance (all from RACMO2/GR, (Ettema et al., 2009; van Angelen et al., 2012)), $\rho_{ice} = 0.917$ kg/m$^3$, and $\rho_0$ is the temperature-dependent density of new firn before the formation of ice lenses. The density of the new firn is calculated from the following empirical relationship: $\rho_0 = 625 + 18.7T_f + 0.293T_f^2$ and $T_f = TMA + 26.6SIR$, where $T_f$ is the firn temperature at 10 m depth and TMA is the mean annual temperature (Reeh et al., 2005).

Typical elevation errors for crossover areas higher up on the ice sheet, involving some one hundred laser points observations, are about $\pm 0.02$ m (Schenk and Csatho, 2012). This compares well with the individual error of a laser point under ideal conditions. At lower elevations, errors increase and reach values of $\pm 1.0$ m or even larger, because of increased slope and rough-
ness (due for example to crevasses). For and temporal changes in the surface shape during the observational period. The error of the firn densification model, we assume an error of 0.005 rate is 0.001 m/yr increasing to 0.01 near the ice divide, increasing to 0.03 m/yr at the ELA (Sørensen et al. (2011), Fig. 4f). This results in a 0.012–0.024 m total error during the ICESat–ICESat mission.

In addition to the GIA, vertical bedrock motion includes the elastic crustal response to contemporary ice sheet mass changes (e.g., Khan et al. (2010), Sørensen et al. (2011)). Uplift rates measured by the Greenland GPS Network (GNET) significantly exceed the predicted GIA rates, indicating that the present-day vertical crustal deformation is dominated by this elastic response (Bevis et al. 2012). Although the magnitude of the elastic response can reach a few cm/yr in the coastal regions, we have not attempted to remove it, because of the low temporal reconstruction of current best reconstructions (3-year averages, personal communication, Abbas Khan, 2014) and the lack of error estimates.

3.2 Spin-up and model inputs

Assimilating altimetry data into a forward transient ice flow model presupposes that the model itself is spun-up in a way that

Since our assimilation method does not adjust initial conditions of the model, we have to rely on a spun-up model state which more or less closely matches surface altimetry observations for the time period considered. Indeed, the success of inverse methods applied to nonlinear problems often relies, in practice, on initial guesses of the independent variables that yield states that are not too far from observations. However, as demonstrated by the wide range of model outcomes in the SeaRISE experiments (Nowicki et al. 2013a–b), spin-ups are very difficult to calibrate. One approach is to run long-term paleo-reconstructions of the GIS in ways that try to match present-day observations (Huybrechts and Oerlemans 1988; Ritz et al. 1997; Pollard et al. 2005; Pollard and DeConto 2009; Greve 1997a). This approach is usually biased towards conservation of mass, where the diagnostics of choice is the ice thickness. It also usually relies on lower-order ice flow models, such as the Shallow-Ice Approximation (Hutter 1982), which
are computationally efficient, but tend to lead to large misfits to observed surface velocities.

Another approach is to rely on what has sometimes been described as "instantaneous spin-ups", in which inversion methods are used to try to capture the dynamics of ice flow at present time. This involves inferring basal friction at the ice/bed interface in order to match present-day observed surface velocities \( \text{[MacAyeal, 1993; Morlighem et al., 2010; Arthern and Gudmundsson, 2010]} \). However, this approach relies on a steady-state thermal \text{and mechanical} regime for the ice sheet, which is not realistic, and usually leads to lumping any mismatch between model and observations into the inversion itself. In addition, as demonstrated on Nioghalvfjerdsfjorden Glacier (hereafter referred to as 79 North), artifacts in the interpolation of bedrock data can lead to ice-flux divergence anomalies, which are not physical \( \text{[Rasmussen, 1988; Seroussi et al., 2011]} \) over time scales of 10 to 50 years. This can be mitigated using a mass-conserving (MC) interpolation approach \( \text{[Morlighem et al., 2013a]} \), allowing for transients that spin-up in ways that match present-day ice velocities and ice-flux divergence. Here however, we are interested in variations in surface heights that are small and could easily be confused for residual ice-flux divergence anomalies that remain even after implementation of the MC approach.

We therefore opt for the different approach of combining both spin-up methods. We carry out an adjoint-based inversion of the basal friction coefficient (we refer the reader to Morlighem et al. (2010); Larour et al. (2012c) for details on the implementation of this inversion within ISSM) using a MC bedrock of the area \( \text{[Morlighem et al., 2013a]} \), followed by a relaxation of the ice sheet/ice shelf over a period of 50,000 years \( \text{(in which forcings are kept constant)} \), until the NEGIS ice volume stabilizes. The climate forcing is constrained by a constant SMB taken equal to the average value between 1971 and 1988, when the GIS was considered more or less in steady-state balance \( \text{[Rignot et al., 2008]} \). This is then followed by a forcing of the ice sheet evolution starting from the Little Ice Age (LIA) in 1840 to the start of our assimilation, in 2003, using the SMB time series from Box (2013). This ensures that our spin-up does not exhibit ice-flux divergence anomalies, matches closely the present-day observed surface velocities \( \pm 100 \text{~m/yr over the whole basin, } \pm 300 \text{~m/yr at the grounding line} \), and responds to variations in climate forcing over the last 173 years.

The ice boundaries for the NEGIS domain are determined by the position of the ice divide (as
determined from the gradient direction of ice sheet surface topography, see Fig. 3f) and ice/ocean as well as ice/rock boundaries from the GIMP project (Howat et al. 2014). We also make sure, given the bedrock is computed using the MC approach (Fig. 3b) that the domain covers the extent of the 2008 InSAR surface velocities, as shown in Fig. 3f, including 79 North and Zachariae–Zachariæ Isstrøm’s ice shelves. The initial surface elevation for the domain (prior to relaxation) comes from the Howat et al. (2014) DEM, which covers the 2003 to 2009 period (Fig. 3a). The resulting ice thickness is shown in Fig. 3c, from subtracting the MC bedrock to the surface height. SMB for the 1971-1988 period comes from the Box (2013) time series (Fig. 3d). $\alpha$, the basal drag coefficient used for the entire length of the relaxation period, as well as the 2003 to 2009 run, is inverted from the 2008 InSAR surface velocities (Fig. 3e). The underlying mesh for the FEM model (Fig. 3f) comprises 1409 elements, for a resolution ranging from 70 km near the ice divide to 30 km at the ELA and 5 km on the 79 North and Zachariae–Zachariæ Isstrøm ice shelves. The altimetry data is interpolated onto the mesh vertices using a linear interpolation algorithm, between February 2003 and September 2009 when continuous data is available.

Starting 2003, the model is run at a two-week time step, in order to coincide with the time-sampling of the surface altimetry observations, and the cost-function $J$ is computed for the entire 2003-2009 time period. The inversion is carried out twice, once for $M_S$, and once for $\alpha$. A simultaneous inversion for both parameters would be more efficient and realistic, however, the ISSM framework does not yet possess the capability to do so, but this type of inversion will definitely be the subject of future studies. Because the model spin-up does not reach a configuration that matches the altimetry time series within a $1\sigma$ standard deviation, we are still forced to adjust the overall mean of the entire altimetry time series so as to center it on the modeled surface height in 2006. A spatially variable time-mean bias is removed from the altimetry observations, corresponding to the difference between the 2006 DEM and the spun-up surface height (Fig. 4). This implies that we are here interested in what corrections have to be applied to $M_S$ and $\alpha$ to match short-term transient variations in surface height. As to longer term trends, we cannot match them, as they depend on assimilating data over much longer time spans, which is not feasible as comprehensive altimetry data coverage on NEGIS prior to 2003.
4 Results

Gradients computed for the temporal inversion of $\alpha$ and $M_s$ exhibit high variability both in space and time (Fig.5), showing that best-fit to observations can only be improved by varying forcings over the entire space and time domain. This is expected given the quadratic nature of the cost function relied upon for the temporal inversion. Indeed, for our cost function (7), the gradient to for example $\alpha$ will be of the form:

$$\frac{\partial J}{\partial \alpha} = \frac{1}{S \Omega} \frac{1}{T} \int_{t=0}^{t=T} \int_{\Omega} \frac{\partial s}{\partial \alpha} (t)(s(t) - s(t)_{obs}) d\Omega dt$$

(11)

showing that the gradient is driven by the misfit $s(t) - s_{obs}(t)$, which is itself time-space variable.

Peaks in the magnitude of both gradients are highly localized, with for example $\partial J/\partial \alpha$ exhibiting clear peaks between 2003 and 2009 over Storstrømmen, which is a surge-recovering glacier (Reeh et al., 1994) where we indeed expect large variations in surface height. Such peaks can reverse sign in time, as is the case for $\partial J/\partial \alpha$ 90 km upstream from Zachariae–Zachariae Isstrøm’s ice front, which is positive in September 2003 and then turns negative starting June 2006. This is in contrast with vast expanses of the NEGIS domain where gradients can be largely constant in space and time. For example, $\partial J/\partial \alpha$ and $\partial J/\partial M_s$ are almost nil 100 km upstream of 79 North and Zachariae–Zachariae Isstrøm. For $\partial J/\partial \alpha$, this can be largely explained by the initial misfit to observations is smaller inland (Fig 8d), which according to (Eq 11) implies that the gradient in the same area will be relatively small. For $\partial J/\partial M_s$, the lack of variation inland suggests that the overall trend in the forward model captures the inland surface height variations realistically, meaning that corrections to the
time series will not be significant, and no obvious bias is exhibited by our forward model. Over the mountain ranges between Storstrømmen and Zachariae Isstrøm, variations in $\partial J/\partial M_s$ are high both temporally and spatially, suggesting that the time series of SMB has a complex signature that may not be fully captured by the SMB forcing. In particular, $\partial J/\partial M_s$ is positive in September 2003, suggesting that improvements in our best-fit to observations will be achieved by decreasing SMB over the mountain range. The situation reverses, with $\partial J/\partial M_s$ turning negative in June 2006, pointing to the need for increasing SMB over this time period. In December 2009, the situation reverses again.

Overall, the variability in both gradients matches closely follow the variability in the ICESat-1 surface height time series (Fig.2). This clearly implies that our model lacks the intrinsic variability required to match observations. In addition, there is a clear demarcation line in the gradient, which runs perpendicular to flow, that coincides with an abrupt transition in ice thickness (see Fig.3, 100 m contour in black) across the entire NEGIS domain. Upstream of this transition line, gradients become much more uniform and diffuse in space, though short variations in time remain significant. Downstream of this line, spatial variations become much sharper, with features developing on the order of 10-20 km. This suggests a controlling mechanism by which sharp transitions in the ice/bedrock interface can reduce the impact of variations in forcings on the overall dynamics of the ice sheet, which could again be related to the fact that gradients above this line are smaller due to an initial low local misfit.

Fig.6 shows the evolution of $J$ during both inversions, over 35 iterations, after which convergence of the optimization is stopped, for considerations that are computational in nature. Both curves clearly demonstrate that corrections in the SMB time series (computed by the temporal inversion) are much more efficient in terms of reducing the overall misfit to observations, than corrections in the basal friction coefficient. This is expected, as SMB is a direct forcing to the mass transport equation (Eq.5), with a clear equivalence between SMB and surface thickening rate, while basal friction is a direct forcing to the stress-balance equations (Eq.1) and (Eq.2), which have no direct bearing on the surface thickening rate. Otherwise stated, it is much easier for the inversion algorithm to match surface heights by adding or subtracting mass directly from the ice column thickness (which is what SMB captures) than by modifying the state of stress.
at the ice/bed interface. The difference in convergence between both methods varying \( \alpha \) or \( M_s \) is significant, with the SMB inversion reducing misfit \( J \) by 68\%, and the basal friction inversion reducing misfit by 14\%.

This is in line with how observed surface heights are matched by the model at locations I and II (see Fig. 1). The first location correspond to the trunk of 79 North, while the second location corresponds to Zachariæ Isstrøm, near the grounding line. Both locations are in areas of enhanced ice flow. At location I, the inversion increases basal friction over the entire ICESat-1 ICESat time period, and the modeled surface height increases by approximately 20 cm starting 2007 (see Fig. 7a and b). The resulting improvement over the initial modeled surface height is not obvious however, and points rather to an increase in the misfit. This increase is localized, and points to an intrinsic inability of the model to match surface heights at this location through variations in basal friction. For location II however, basal friction is decreased by the inversion between 2004 and 2008, and results in a much better fit to local surface heights, with significant improvement between the initial and final modeled surface height. At this location, the model is therefore capable of correcting the basal forcing appropriately to match observations. The local nature of the improvements is confirmed in Fig. 8, which shows that location I is in an area of increase of the misfit to observations, while location II is in the area that sees the most improvement.

For the SMB inversion at location I (Fig. 7c and d), an increase in SMB slightly prior to 2006, and a significant decrease by almost 30 cm/yr after 2007 is modeled. The maximum decrease occurs around summer 2008. The resulting modeled surface height matches observations significantly well, with a decrease in the modeled height reaching up to 60 cm in year 2009. The situation is very similar for location II, with however one difference, the magnitude of the SMB correction, which is very large at location II, with significantly more melting modeled by the inversion. Overall, the SMB inversion improves the best-fit to observations much better than the basal drag inversion, as confirmed by Fig. 8. An interesting point is that the structure of the correction tightly matches the structure of the underlying SMB time series itself. Indeed, SMB is corrected mainly between peak summer rates, with the peaks themselves being preserved. Almost no correction to the summer values is detected, which is interesting given that
the time step used for the transient runs is 2 weeks, which is short enough to allow the inversion
to capture and correct peak summer rates. This suggests that modification in the summer peak
magnitude does not profoundly impact the best-fit to observed altimetry time-series, and that
the average summer-to-summer value is what critically controls the inter-annual variability in
surface heights. **It must be noted however that the inversion also predicts negative values for
the melt-rate throughout years 2005-2008, which is highly unrealistic. This is certainly due to
the fact that our SMB inversion does not constrain the variations of $M_s$ within a certain error
range, as should be the case. This is certainly an aspect of our method that will be improved in
further studies, in order to account for realistic variations in the model inputs.**

Overall, the best-fit to observations is improved by the inversion. However, locally, the im-
provement can be widely different. Fig 8 shows how $J_T(x,y)$ (cf. Eq. 9) is spatially distributed
across the entire basin after inversion and what the resulting improvement $\Delta J_T(x,y)$ is. As
expected, $J_T$ is much lower after inversion of SMB as opposed to basal friction (Fig. 8b vs
e respectively). In terms of local improvement, $\Delta J_T$ is largest near the coastline, while it is
more diffuse across the entire basin. For SMB, a greater decrease occurs near the main trunks
of 79 North and Zachariae Zachariae Isstrøm, but the initial value of the misfit being also much
higher, this still results in large misfit values near the coastline. For basal friction, the descent is
much more difficult for the trunks of the two glaciers, with clear decreases of the misfit near the
grounding line, but small increases directly upstream of the grounding line. Indeed, as demon-
strated by Fig 7 and confirmed on Fig 8, for location I the local misfit does increase for the
basal drag inversion. This is compensated by large decreases near the grounding line and ice
shelves of 79 North and Zachariae Zachariae Isstrøm. Overall, a significant amount of misfit
still remains at the coastline, where both inversions seem to be unable to further accommodate
for short-term variations in surface height. Near the ice divide, the SMB inversion improves
the best-fit most, which can be explained by the uniformity of the $\partial J/\partial M_s$ gradient in this
area, allowing for a fast descent of the inversion algorithm. Of course, the best-fit to observa-
tion depends on whether the convergence has been reached, and further improvements might
be expected if the number of iterations is increased. This is especially true for the SMB inver-
sion, which exhibits variations in $J$ iteration on iteration of 0.6% for each iteration, after 35
iterations, as opposed to 0.15% for the basal friction inversion. However, these values are significantly below the 1% threshold, and we therefore do not expect large differences compared to a fully converged inversion.

5 Discussion

Our temporal inversion allows for the determination of forcings that best-fit observations and that satisfy the physics described in the forward model. This is to our knowledge the first time this approach has been successfully carried out using an SSA based transient ice-flow model, without relying on arbitrary tuning of surface and basal forcings. Our results clearly demonstrate that high variability in the model forcings is necessary to reproduce NEGIS surface altimetry from 2003 to 2009. Such variability is both exhibited in the inversion of $M_s$ and in the inversion of $\alpha$, which suggests that our forward model lacks internal representation of such variability, and that enhancing our representation of boundary conditions $M_s$ and $\alpha$ in the forward model is therefore necessary.

At the surface, $M_s$ accounts for changes in the altimetry in terms of ice-equivalent mass (see Eq. 5). This is compatible with our ice flow model, which is based on the assumption of incompressible ice-flow, in which ice density is constant and equal to $917 \text{ kg/m}^3$. In order for the altimetry to be converted from the original surface to an ice-equivalent surface, two firn-densification models were therefore used. In the percolation-wet snow-superimposed ice zones, the firn densification model from Reeh et al. (2005); Reeh (2008) was used, which includes the effect of ice lensing and is forced by the RACMO2/GR data (Ettema et al., 2009; van Angelen et al., 2012). Above the ELA, the firn densification model from Lucas-Picher et al. (2012), forced by HIRHAM5 climatologies was used, which accounts for densification through pore-space closure. Both models depend on atmospheric constraints such as accumulation rate, surface temperature and surface snow-density. By relying on ice-equivalent thickness, it is therefore difficult to attribute which component of the variability exhibited in $M_s$ is due to the variability in the climate forcings used in the firn densification formulation (such as surface snow density, ice-lens content, accumulation rate, surface temperature, etc.). Therefore,
while our approach clearly demonstrates that SMB time series for the area need to be corrected for, it also shows that without clear representation of firn-densification processes in the forward model, we cannot improve our understanding of which atmospheric and/or surface processes is most responsible for the surface height signature of NEGIS. It is therefore our intention to refine our approach in further studies, towards temporally inverting for surface snow density, surface temperature and accumulation rate, with the goal of understanding which one of these processes is responsible for most of the variability observed on NEGIS. Ultimately, the hope is that inclusion of a firn-densification representation in the forward model will lead to increasingly smaller corrections required on the corresponding forcings, thus ensuring that the model itself intrinsically captures the observed surface height variability.

At the ice/bed interface, the basal drag coefficient exhibits high spatial and temporal variability which could be due to the underlying basal hydrology. Indeed, though a clear relationship has to our knowledge never been demonstrated, calibrated or measured between basal stress $\tau_b$ (or driving stress $\tau_d$) and sub-glacial water pressure $w$, empirical arguments such as in Alley (1989) suggest a relationship of the type $N_{eff} = k_n \frac{\tau_d}{w}$ where $k_n$ is a basin-scale constant parameter. Because driving stress and basal stress are closely related through the stress balance equation, such relationships hint at a direct link between a highly variable drag coefficient and water pressure. Indeed, assuming this type of relationship holds, our approach can quantify variations in water pressure under the entire basin that can explain the observed variations in surface height. By a reasoning similar to our approach for surface mass balance, our results demonstrate the need for further integration of hydrological models in our forward model so that we can improve our understanding of how surface height variability can be generated by the water pressure forcing.

By design, our inversions were carried out independent of one another, which makes it difficult to attribute to either basal friction or surface mass balance the inferred variability in surface height. The fact that convergence is reached much faster, and much more efficiently for SMB than for basal friction is a strong hint that most of the variability is probably atmospheric in nature; however, we cannot disregard entirely the variability in basal water pressure. Indeed, recent studies suggest strong links between water pressure and surface melt water draining.
through moulins and lakes, which can be seasonally driven (Alley et al., 2005; Luthje et al., 2006; Box and Ski, 2007; McMillan et al., 2007; van der Veen, 2007; Tedesco, 2007; Shepherd et al., 2009; Palmer et al., 2011; Tedesco et al., 2012). Another issue our inversions raise is the fact that surface altimetry is strongly biased towards inferring changes in surface mass balance, which if we are to improve our understanding of variability and trends in basal hydrology, presents a challenge. Indeed, in order to invert for variations in basal friction, our observable should be surface velocity, as it is directly linked to basal stress through Eq.1 and 2, and plays a similar role to \( M_s \) in Eq.5. Several studies have demonstrated the usefulness of such an approach for steady-state ice flow model inversions (MacAyeal, 1993; Morlighem et al., 2010; Vieli et al., 2006; Arthern and Gudmundsson, 2010), and our results suggests this extends to transient ice flow models as well. Here, we propose that as previously alluded to by Heimbach and Bugnion (2009) and explored in Goldberg and Heimbach (2013), a combined approach be entertained, in which both surface velocity and height be used to invert for the state of the ice at the ice/bedrock interface and at the surface. This puts serious constraints on the rate at which surface velocities from SAR platforms should be collected, but the emergence of satellites such as TerraSAR-X or Sentinel, which can provide high-repeat pass observations, in combination with continuous coverage from altimetry by CryoSat-2, submeter resolution stereo imaging from Worldview-2 (Shean et al., 2012) and in the coming years ICESat-2, shows a high-degree of promise.

Some improvements to our methodology, which we are currently working on, should alleviate some of the issues regarding attribution of surface height signatures to surface or basal processes. Indeed, our inversion is not constrained by error margins in both our model forcings (in particular SMB) or model diagnostics (surface altimetry). By introducing such margins, we would ensure that our forcing corrections remain within the bounds of what is realistic. Indeed, it is highly probable that our cost-function decrease for the SMB inversion is too drastic, and generates corrections that are too large to be acceptable within the uncertainty range of the time series from Box (2013). For basal friction, it is very difficult to assess the error margin on the initial time series. However, provided a basal hydrology model is included in the forward model, a better quantification of the uncertainty in the underlying hydrological model should be
possible, which should result in a better quantification of the uncertainty in the computation of the basal drag coefficient itself.

Both inversions provide good results inland, where misfit is lowered significantly. Near the coastline however, misfit remains significant (Fig.8). The coastline is a very mountainous area, with few outlet glaciers (79 North, Zachariae–Zachariae Isstrøm, Storstrømmen) that are in contact with the ocean. For these outlet glaciers, the misfit can probably be attributed to a lack of representation of ice/ocean interactions. Indeed, melting rate under the ice shelf is taken constant during grounding line retreat, which does not take into account variations in sub ice-shelf cavity circulation. For the remainder of the area however, in the mountain ranges near the coast, high misfit is still observed. Given that ice velocities are negligible there, the misfit must be attributed to variations in SMB that are not captured in the initial forcing. This suggests large corrections are still required in the SMB local to these high-altitude areas. This could suggest two things: 1) that the altitude/lapse rate parametrizations need to be improved; 2) that our inversion needs to be locally and temporally refined in these specific areas. Indeed for the latter, our gradients computed in Fig.5 provide a basin-scale vectorial direction along which the steepest-descent algorithm optimizes the cost-function. However, smaller areas of NEGIS could be considered for the SMB inversion, for example those areas which exhibit high hypsometry only.

6 Conclusions

We presented a new data assimilation system within the ISSM framework that is capable of assimilating surface altimetry data from missions such as ICESat–ICESat into reconstructions of transient ice flow. This system relies on algorithmic differentiation at its core to compute gradients of diagnostics (such as a cost-function between modeled and observed surface height) with respect to model forcings. An application to the North Eastern Greenland Ice Stream was provided, where surface mass balance and basal friction forcings were temporally inverted, resulting in significant improvements in the best-fit to observations. This new approach allows for a better understanding of which processes can be characterized by altimetry,
and illustrates the need for combining different datasets such as altimetry and satellite-derived surface velocities into inversions of basal friction and surface mass balance. It also enables a better quantification of the contribution of each forcing to the model best-fit to observations, and a better understanding of which type of physics are currently missing from transient ice flow models in order to better characterize the important intra and inter-annual variability in surface heights. Our results also demonstrate that large spatial and temporal variability is required in model forcings such as surface mass balance and basal friction, variability that can only be explained by including more complex processes such as snowpack compaction at the surface and basal hydrology at the bottom of the ice sheet. Our new approach, once combined with estimates of errors in the model inputs, should allow for a better identification of which underlying processes are responsible for specific signatures in the observed surface altimetry. This approach is indeed a first step towards assimilating the wealth of surface altimetry data that is currently available from EnviSat, ICESat-1, ICESat, Operation IceBridge and CryoSat-2, and that will be available in the near future with the launch of ICESat-2.

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Fig. 1. Map of the study area, North-Eastern Ice Stream, Greenland. InSAR surface velocities from Rignot and Mouginot (2012) are displayed overlayed over the MODIS Mosaic of Greenland (MOG) image map (Haran et al., 2013). The data is projected using the NSIDC Sea Ice Polar Stereographic North projection (EPSG:3411) with central meridian at -45° and standard parallel at +70°. Locations marked in yellow, I and II are used in Fig. [7].
Fig. 2. Annual elevation change rates from ICESat-1 satellite altimetry between Fall 2003 and Fall 2008, computed from a polynomial approximation of the elevation change history as described in Schenk and Csatho (2012). Locations for which SERAC processed altimetry data is available for the entire time series are indicated by black dots. Elevation changes are computed for the balance years starting on September 1 and ending on August 31 of the following year.
Fig. 3. a) 2007 surface height from Howat et al. (2014). b) bedrock elevation inverted using the mass-conserving approach (Morlighem et al. 2011, 2013a), existing bedrock data from Thomsen et al. (1997); Christensen et al. (2000), IceBridge MCoRDS ice thickness data (Allen 2011) and InSAR surface velocities from Rignot and Mouginot (2012). c) ice thickness inferred from a)-b). In black, 1000 m contour level. d) 1971-1988 averaged yearly surface mass balance $M_s$ (m/yr) from Box (2013). e) drag coefficient $\alpha$ ((m/s)$^{-1/2}$) inverted using 2008 InSAR derived surface velocities from Rignot and Mouginot (2012) (see f). f) model mesh, superimposed on 2008 observed surface velocity (m/yr) derived from InSAR (Rignot and Mouginot 2012).
Fig. 4. Difference $s_f - s_i$ (in m) between the spun-up surface height ($s_f$) and the reference height for the altimetry data, taken to be the Howat et al. (2014) surface DEM in 2006 ($s_i$). This spatially variable time-mean bias is removed from the observations to reach a better adequation between the model spin-up and the altimetry data used in the inversion.
Fig. 5. a), b) and c): gradients $\frac{\partial J}{\partial \alpha} \left( m^{3/2}.s^{-1/2} \right)$ of the cost function $J$ with respect to the ice/bed friction coefficient $\alpha$ in respectively, September 2003, June 2006 and February 2009. d), e) and f): gradients $\frac{\partial J}{\partial M_s} \left( \text{yr} \right)$ of the cost function $J$ with respect to the surface mass balance $M_s$ in respectively, September 2003, June 2006 and February 2009.
Fig. 6. Evolution of the cost function $J$ during the inversion of $M_s$ (in red) and the inversion of $\alpha$ (in black), over 35 iterations.
Fig. 7. Improvement in the best-fit between modeled surface height compared to and the altimetry record after inversion of basal friction $\alpha$ and surface mass balance $M_s$ for the two locations indicated in Fig. (3f) corresponding to the center of 79 North (frames a), (b), (c) and (d)) and a location near the grounding line of Zachariae–Zacharias Isstrøm (frames (e), (f), (g) and (h)). Frames a) and e) show the difference between the original time series $\alpha_0$ of friction (in black), and the inverted one $\alpha$ (in red). Frames b) and f) show the improvement in the modeled surface height (in red) best-fit to observations (in blue) compared to the original model (in black) and the better fit to observations (in blue). Frames c) and g), d) and h) show similar results as for a) and e) and b) and f), with surface mass balance $M_s$ being the quantity inverted for instead of $\alpha$. Errors in observed surface height time series for locations I and II, which are below the ELA, depend on the SERAC processing only (ice is assumed fully dense, with no firn compaction involved), and are estimated respectively at 8 cm and 1.2 m.
Fig. 8. a) and d): localized misfit $J_{T0}(x,y)$ as defined in Eq. (9) before inversion of surface mass balance $M_s$ and basal friction $\alpha$ respectively. b) and e): localized misfit $J_T(x,y)$ after inversion. c) and f): corresponding decrease in localized misfit $\Delta J_T(x,y) = J_T(x,y) - J_{T0}(x,y)$ before and after inversion.
Discussion Paper

References


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