We are grateful to the referees for their positive evaluation of our work and particularly for the detailed comments. We made corrections in the manuscript according to the referees’ minor comments. In the following we give more detailed answers to their questions. The revised version of the manuscript will be submitted when we will have the answers of the reviewers.

Anonymous Referee #1:

This manuscript details a model simulating shortwave radiative transfer for melt ponds on the surface of Arctic sea ice. The paper is of interest to TC readership and describes a model that appears sound and well tested. The language is a bit awkward in places (see minor comments below), but I do think it is generally readable.

Thank you. As for our English, we did our best and particularly mindfully considered your minor comments and made appropriate corrections in the text. For the final version we will have an additional round of correction by our native speaking coauthors. Besides, there will be English copy-editing by the editorial staff at the final stage.

My only major comment on the presentation is that p. 16 line 16 states that three independent parameters are required for this model: pond depth, ice substrate thickness, and ice transport scattering coefficient. I agree. The results presented in Table 2 show the first two parameters. What is assumed about the third one? There is no information discussed in the manuscript that would suggest what values were inferred for the ice transport scattering coefficient. Almost all of the comparisons between model and observation show remarkable fidelity. It seems so remarkable, that I wonder what range of transport scattering coefficients are used, and whether there is some vertical variability allowed in the ice layer beneath the pond water for that coefficient? If I understand correctly, the model is inferring an optical depth $\tau$ so an assumption must be made about the inherent optical properties in order to retrieve the physical depth of the ice? What is that assumption?

You are absolutely right that just the optical depth, rather than the geometrical one, determines the reflectance. They are related by Eqs. (42). We consider all three parameters, $z$, $H$, and $\sigma_t$, as independent ones. We vary all of them independently when fitting spectra and don’t make any additional assumptions about $\sigma_t$. (Except vertical variability). Of course, we don’t have enough information to retrieve the vertical profile of $\sigma_t$, so we assume that we retrieve some constant effective value for a layer). Thus, all these three values are retrieved for every spectrum. In Table 2 we show only two of them just for comparison with the \textit{in situ} measured values of $z$ and $H$. This information will also be added to the manuscript. However, nobody measures $\sigma_t$, so we don’t show its values. But we added the retrieved values of $\sigma_t$ for the light and dark portions of the SHEBA pond (see the last paragraph of Sec. 4.3), where they are important for calculation of the scattering coefficient by bubbles.

Minor comments:

p.1 line 9, 19: “large part” and “large fraction” are not very specific
We omit general references and put: “up to 60% on multiyear ice according to Maykut et al. (1992) and up to 80% on landfast ice according to Langleben (1971).”

p.1 line 24: “nowadays” colloquial

Changed: “in light of the environmental changes observed recently”

p.2 line 5-6: “Makshtas and Podgorny give a formula for pond spectral albedo at direct incidence only; they do not consider the angular distribution of the reflected light.” This sentence is a bit confusing. I understand that M&P give a formula for pond albedo only for direct incidence, but I don’t see why that relates at all to an angularly-resolved description of the reflected field.

We changed the phrase to:
“Besides, the question of the angular distribution of light reflected by a melt pond is still open.”

p.2 line 15: “banner of the ice grains presence” makes no sense

Changed to “evidence of the ice grains presence”

p. 2 line 16: “common deficient information” makes no sense

Changed to “when the incident angle is unknown”

p.4 line 1: define ‘AW’
p.5 line 3: define ‘WI’

Definitions are added in p.3, l.27.

p.6 line 6: please supply a reference for the definition of ‘transport scattering coefficient’

We added the references Davison, 1958 and Chandrasekhar, 1960

p. 6 line10: does ‘very elongated’ phase function mean ‘very forward peaked’ phase function? I don’t believe ‘very elongated’ is commonly understood. I think the authors are attempting to convey the idea that a smaller scattering coefficient and lower |g| can be used to describe the apparent optical properties of a medium with large scattering coefficient and/or high |g|.

Yes, you are absolutely right. Corrected.

p. 6 line 22: mirabilites and hydrohalites. . . should be mirabilite crystals and hydrohalite crystals

Corrected.

p.6 line 26-28: If the highly scattering surface layer isn’t being considered here, then
what is being considered?

We tried to specify a little: “We do not consider here the highly scattering surface layer that is formed on top of sea ice during the water drainage process and is commonly referred to as ‘white ice’.” Hope it’s clearer.

p.6 line 28: Statement that air bubbles in sea ice are mostly spherical needs a reference.

We added Gavrilov and Gaitskhoki, 1970; Mobley et al., 1998; Light, 2010.

p.7 line 3: is exponent +1.24 or -1.24?

Corrected to –1.24.

p.8 line 20: sloppy notation, with the ‘t’ used as a subscript on the left hand side of the equation and as a superscript on the right hand side, but both mean the same thing.

We hope this notation will not confuse our readers. After all, these sub- and superscripts are not the tensor indices where their position is principal.

p.11 line 11: ‘extra-terrestrial solar irradiance’ I think is more commonly called ‘top-of-atmosphere irradiance’?

Both terms are widely used. As for our experience, the term ‘top-of-the-atmosphere’ is more frequent for the Earth reflected radiance, while ‘extra-terrestrial’ for solar light.

eqn 49: it is confusing that both A and alpha are used for albedo

We replaced \( \alpha \) by \( A^{blue} \).


These are the dates of the cruise. We added: “The melt ponds were observed in August.”

p.13 line 11-14: The description here lacks detail. I assume the fiber optic probe coupled to the ASD is used to view light reflected by the Spectralon plate, but this isn’t adequately described. The phrase “served as a diffuser” doesn’t completely describe how the Spectralon plate was employed.

We added: “A sensor measures the light signal supplied by a fiber optical probe, which collects light reflected by a 10x10 cm\(^2\) Spectralon white plate.”

p.13 line 31: what does ‘open’ mean here? No ice skim?

Yes. We put an explanation in the beginning of Sec. 4.1. “The melt ponds were observed in August, being both open (with no ice skim) and frozen over (with a skim of ice), sometimes snow covered.”

p.14 line 31: the spectral albedo was taken every 4 days?
Yes. We put: “The spectra were taken every four days during this period. The spectra processing results are shown in Figs. 12 and 13.”

Fig 5 The angle of incidence is stated in the text, but needs to also be stated in the figure caption.

Done.

Fig 6 Where did these spectral curves come from? There needs to be some data attribution.

These spectral curves are modeled for the typical values. We put a phrase “Typical spectral albedo of melt ponds, snow, and white ice, calculated for the following parameters:

Fig 7 caption should include information (from text) that these all had 2-3 cm layer of ice on top.

Done.

Fig 7 I am surprised at how high the albedo is at blue wavelengths! Could this be due to the frozen surface? If so, then that would contradict the statement p.13 line 21. I would expect the peak albedo at blue wavelengths for unfrozen melt ponds to be somewhere in the range 0.1 to 0.5, at most.

Actually, we cannot be sure that high albedo values do not come from the frozen surface. If an ice skim contains a lot of air bubbles, it can increase the reflectance, but in this case it becomes indistinguishable from the ice substrate. So the optical thickness retrieved is the total thickness (skim + substrate). As we wrote, our model does not consider such cases. The statement p.13 line 21 only states that a layer of transparent ice does not change pond reflection. On the other hand, there is no restriction of 0.5 for open pond albedo. To be objective, we put the phrase in p.13, line 30: “The albedo values are extraordinarily high. This could be related with the fact that the ponds are frozen over with a 2-3 cm layer of ice on top.”

Would be useful to show all the panels in each cluster (Figs 7, 8, 9 each a cluster) on the same vertical (albedo) scale.

The plots are quite small, we think it’s better when the drawings take all the scale.

Also, captions for Figs 7, 8, 9, 10, 11 need to contain information about the general locations of each series.

We put in the text about the Polarstern cruise: “The stations, where ponds were observed, were located from 84°3N, 31°7E to 82°54N, 129°47E. For more information about the cruise, see Boetius et al. (2012) and Istomina et al. (2016).”

For Barrow and SHEBA the locations are given: Chukchi and Beaufort seas.

Fig 8 If these ponds were heterogeneous, then the exact location of the albedo measurement matters! Can this location be shown?
The exact point of the measurement can be seen in the photograph, where a person is taking observation from the light portion of the pond. Unfortunately, there is no photo for the dark one.

Fig 12 Caption says ‘on June 3’, but I believe was July?

Of course, July. Thank you.

Anonymous Referee #2:

The manuscript describes a new numerical model to calculate the spectral reflectance of melt ponds on Arctic sea ice, mostly determined by three independent variables. The authors find good agreement between simulated and observed spectra from in-situ measurements during three different field campaigns. This allows them to derive water depth, under pond ice thickness and transport coefficients for each of the ponds. Given the ongoing changes of the Arctic sea ice cover towards longer melt periods and increasing melt pond fractions, the manuscript describes a timely topic, which is well suited for publications in The Cryosphere. Over all, I suggest publication after minor revisions, which mostly comprise some additional discussion and sharpening of the main conclusions.

Thank you.

General comments:
- It is not clear to me what the NEW elements of this model are, compared to existing models and theoretical approaches. It seems that most relations and assumptions are taken from existing studies. Since this is a mostly methodological manuscript, the following aspects need to become obvious:
  - What are the additional and new insights into radiative transfer of melt ponds?

Actually, we don’t know any existing models or theoretical approaches that relate the pond reflectance to its physical characteristics. The work of Makshtas and Podgorny relates the pond albedo to the albedo of its bottom only. In our work we show how to obtain the spectrum of the pond bottom albedo through the radiative transfer characteristics of under-pond sea ice. To do so we use the approaches, developed by the authors for light scattering by non-spherical particles within the WKB approximation (Malinka, 2015) and for radiative transfer within the two-stream approximation (Zege et al, 1991). We show which particular parameters determine the pond bottom albedo. These parameters are really the transport scattering coefficient and ice thickness. Besides, we pay particular attention to two more points: the bi-directional reflectance, which is of great importance for remote sensing techniques when processing satellite data, and the atmospheric correction of in situ measured data, which is hardly made by anyone for in situ measurements. As far as we’re concerned, we think that all these points are stated in the Introduction. Also, according to your advice, we added these points to the Abstract.
o How can or should this model be used in future (the outlook at the very end is rather unspecific and too general)?

o What kind of scientific merit do the authors expect from this and following studies (applications of the model).

Of course, we cannot predict all possible merits. But some applications are obvious: such a model is absolutely necessary for satellite data processing in remote sensing of Arctic ice. Particularly, this model has served as a basis for the MPD (Melt Pond Detector) algorithm for melt pond fraction and sea ice albedo retrieval from MERIS data (Zege et al., 2015).

- The authors conclude that only three independent parameters are needed to characterize melt ponds and thus to retrieve an appropriate optical characterization from them. They do discuss and show results of pond depth and substrate thickness, but I am missing an analysis and more discussion and details on the transport coefficient. In that respect, the role of the three main parameters should be discussed in the discussion and be concluded at the end of the manuscript. How do they impact the model (not only in equations) and what sensitivity do we expect and receive?

We consider all three parameters, $z$, $H$, and $\sigma_t$, as independent ones. All these three values are retrieved for every spectrum. In Table 2 we show only two of them just for comparison with the \textit{in situ} measured values of $z$ and $H$. Nobody measures $\sigma_t$, so we don’t show its values. Additionally, we can add that the transport scattering coefficient is mostly variable due to air bubbles in sea ice. We appended the section dedicated to the dual pond measured in SHEBA expedition with the transport coefficient values for the light and dark parts, which gives the idea of the effect of the transport coefficient on the pond albedo.

- The comparison with in-situ observations show differences of under-pond ice thickness and water depth of 50% and some even significantly higher. I do not follow the argumentation that this is satisfactory, in particular since there is very little discussion about this (see comments below). I consider these differences as more significant than the discussion reveals. In particular with respect to the under-pond (substrate) thickness, which should be the most important parameter to determine pond albedo.

Actually, the most important parameter that determines the pond albedo is the transport optical thickness of under-pond ice $\tau_t$, that is a product of the transport scattering coefficient $\sigma_t$ and ice thickness $H$: $\tau_t = \sigma_t H$. Partially this explains the retrieval error: $\tau_t$ is retrieved with much higher accuracy, however there is no way to compare it with a measured value. There could be also other different sources of error. First, the under-pond ice might not be flat, especially its lower boundary. In this case the optical retrieval gives some average value, while the \textit{in situ} measurement gives a random value taken in some particular point. From this point of view the measurement makes a mistake, rather than the retrieval. The second source can be the presence of some impurities that affect the absorption spectrum. Additional absorption can affect the retrieval of the scattering coefficient and, consequently, of $H$. Besides, there could be other sources of uncertainties, like finite pond size, presence of snow in the receiver FOV, clouds in the sky etc. In view of that, the RMS error of 37% seems to us more than reasonable, especially given the fact that the microwave sounding methods fail absolutely in ice thickness retrieval, when ice is covered with a thin water layer.
Note: I am puzzled about the term “substrate”. Why not under-pond ice thickness?

Thank you for the prompt. As we already mentioned, we are not native English speakers. We have changed this term.

Specific comments:
Abstract: The abstract may be significantly improved by adding more results and a statement that explicitly names the additional benefit and further applications of the model:
- Page1/Line15 (P1/L15): … are examined: What is the result of the examination?

We added: “We find that atmospheric correction is necessary even for in situ measurements. Thus, an atmospheric correction procedure has been used in the model verification”

- P1/L16: several => three field campaigns

Changed

- P1/L17: “good performance” this is rather relative, good in what measure?

How can we measure the adequacy of a model or a theory? This is rather quality, than quantity measure.

- Why are the three main parameters not mentioned in the abstract? How do they perform?

We added some details into the Abstract. Now they are mentioned.

- What does this model stand out for and what is the (likely) future benefit of this study/model?

The model is needed to get and study a quantitative relationship between the physical characteristics of a melt pond and its reflectance. This quantitative characterization will be helpful in retrieving melt pond fraction from space and thereby quantifying the atmosphere–sea ice–ocean heat fluxes relevant for climate research.

Introduction
- Recent studies by different groups show the increasing fraction and importance of melt ponds. Also shifts in melt onset and melt season duration are observed and discussed in various ways. I am missing this aspect in the introduction, while this would add to the motivation of this study and model development.

We added these facts into the Introduction, together with the reference ‘Markus et al., 2009’.

- In addition, there are various approaches to parameterize melt ponds in circulation models of various complexities. This should also be included and could even link to the role of light transmittance into and through sea ice (the remaining after reflection).
This could also well link the introduction to the final part of the conclusions (see comments below)

We added the phrase about light transmittance to the Conclusion

- P2/L4: Include also “water” properties.

If we understand correctly, this comment refers to the sentence “This solution has required the detailed consideration of the inherent optical properties of sea ice, which forms the pond bottom.” If so, we don’t think it is worth including ‘water properties’, because this would mean ‘sea water’ IOPs, which is a very elaborated problem that is very separate from ‘sea ice’ IOPs.

Model descriptions
- This section is most detailed. It could be improved by distinguishing better between existing models and theories and highlighting new ideas and findings.

It is stated in the Introduction: “Subsection 2.1 presents the derivation of the formulas for pond reflectance, given by Makshtas and Podgorny (1996) expanded to various incident conditions.” All other findings throughout the manuscript are original. We do not see how to distinguish better.

- The role of the resulting three main parameters should be highlighted.

These parameters determine the pond spectral reflectance. The coincidence of measured and modeled spectra allows us to state that on more parameters can improve the model and make it closer to reality (unless we see real difference in spectra, which we attribute to some sediments). (we added this to conclusion, also, see below).

Additionally, we added the explanation to the end of Sec. 2.4:
“So, in the absence of pollutants just three parameters determine the pond spectral reflectance: namely, the transport scattering coefficient \( \sigma \) and geometric thickness \( H \) of the under-pond ice and water layer depth \( z \). This statement is confirmed by the coincidence of measured and modeled spectra demonstrated below.”

- It would add value to the manuscript if the model is made available for other users. How is the model implemented? How (numerically) costly are the simulations?

The model is very simple in the implementation, because it is entirely based on analytical formulas. The only numerical cost is the calculation of functions \( f_{in} \) and \( f_{out} \) (integrals in Eq. (14) and (22)). However, these functions can be calculated once for given set of wavelengths and then used as a look-up-table to speed up the simulation. As for the rest, all the formulas are given in the manuscript and can be used straightforwardly.

We added this aspect to the end of Sec.2.1 and to the Conclusion.

Model verification
- P13/L16-19: The realization of the validation and comparison should be described in more detail.
To find the best fit solution we use the multidimensional Newton-Raphson method with the singular value decomposition of the pseudo-inverse matrix. We really think that the discussion of the method lies far beyond the paper scope, but the method name is added to the manuscript. Adding computational details will make the understanding of the work only harder. Also we are sure that the particular method of searching solution doesn’t matter for model verification. It is enough that we find such values of the three pond parameters that give the best fit of spectra in the sense the least squares.

- How did the authors derive that these are the three main parameters. What other parameters were analyzed?

See above our answer about the role of these three parameters. Additionally we can note that refractive indices and absorption spectra of ice and water were not analyzed, because they are fixed, and sediment concentration was not analyzed, because we have no information about polluting substances. So, no more parameters can be analyzed from the point of view of albedo spectrum. Another question is that the transport scattering coefficient consists of the contributions of air bubbles and brine inclusions and thus is determined by their concentrations. Their relationships are considered in detail in Sec. 2.2c and 4.3.

- What about the transport coefficients? How were they studied/discussed?
- How are the thicknesses retrieved?

All three parameters are retrieved in the same manner. They comprise a 3d-vector, which is varied to provide the best fit of spectra. We added this phrase to Sec. 4.1.

- It is a disadvantage that most ponds were not open ponds as it is assumed in the model development. I do see the constrains through the given data set, but this weakens the verification and needs more consideration. Why is there e.g. no thin surface ice in the model?

For the same reason we are also not quite satisfied with the dataset, but that’s what we have. We made computations for the model with frozen surface. Adding a thin ice layer on top changes almost nothing in the results however makes formulas much more tremendous, so we decided not to include them into the manuscript. This overloaded model was formulated in our internal report. At first, we planned to attach this report to the manuscript as a supplement, however the editorial refused it. And we agree with them, because it gives too little new information.

- P14/L14: Add the year (2008) into the main text.

Added.

- Section 4.4 should be the main discussion of the comparisons. This is too short and somewhat superficial.
- Where do these rather large differences of 50% come from? I do see various reasons in e.g. pond depth distributions, non-planar interfaces, footprint of sensors compared to pond properties. But this needs to be discussed in more detail.
- What precision may/can be expected in such models?
What determines the uncertainties? Which of the given assumptions might not be ideal, but what would it mean to adapt this? It is most likely not realistic within this study, but some additional discussion would be useful and interesting for further studies.

Throughout the manuscript, making the derivations, we stated the assumptions we use in the model. Surely, every assumption is some approximation or idealization and any of them can limit applicability and accuracy of results. However, the perfect fit of the measured and modeled spectra is a proof that these assumptions were reasonable.

- With respect to those differences: As discussed, impurities are mostly low in the ponds, so the result is mostly based on scattering (not absorption). In this case, the retrieved spectral shape may be expected to be in good agreement, while amplitude is the main aspect of evaluation. But if then the simulated differences are still around 50% for the under-ice thickness this is somewhat surprising to me. I agree that the RMSE match is quite good if not excellent, but may be not because of the right thicknesses, but other parameters in the model. This should be discussed more.

We think this question is answered in the section ‘General comments’ (the 4th question). (Also note that the mean error for ice thickness is 37%, not 50%).

Conclusions
- Given that ponds may be described by the three parameters: How would future applications look like? What is the main benefit from this conclusion? (P16/L16)

It is just a scientific statement. Actually, reducing the number of key physical parameters down to three is indeed the main benefit.

- P16/L27: This raises the question: How much of the model has been used before and what is new (see above)?

This model was almost fully used in the MPD algorithm described in Zege et al., 2015, but a detailed description hasn’t been published until now. The new modification is that the two-stream approximation is used now instead of the radiative transfer asymptotic formulas for weak absorption. This allows widening the scope of the applicability to significant absorption, what is important in the red and near IR range. The second one is that the scientific justification is given for the sea-ice IOPs and, consequently, to the role of the transport scattering coefficient.

- P16/L30: “can be useful”: This is somewhat vague. How can it realistically be used?

For example, for a better understanding of the Arctic energy budget the quantitative characterization of melt pond reflection is needed. At least, it is needed for satellite retrieval of melt pond fraction.

- The last lines of the manuscript are not convincing to me. How would these improvements be implemented? What are the next applications or which part of these results is most promising. This needs a more thoroughly discussion and a more specific outlook.
The most promising is the relationship between the physical and optical parameters of a melt pond. We think this relationship is needed to study, e.g., the process of ice melting, which is highly determined by its radiative budget.

- The conclusions section misses a conclusion on the uncertainties and deviations from the field measurements (Section 4.4). At the same time, I suggest to highlight that the validation was done against quite a suite of field measurements and variable pond conditions. This is a valuable aspect and could be stressed more. Many studies limit their validation to a single data set (e.g. one field experiment).

We think that most of the facts are performed in the main text. We added the names of expeditions once again to the conclusion.

Table 1
- I think that this is not needed.

The purpose of the table is clarifying for the reader which parameters are variable (and, consequently, are varied in the retrieval) and which are fixed in the model.

Table 2
- The pond code names seem to be an internal coding with almost no use for other studies. Using station names and dates as identifiers that link to field reports, Polarstern station numbers, and Pangaea data sets is suggested.

We put the station number in the case of Polarstern expedition.

- I suggest to re-arrange the columns and group retrieved/measured/difference (absolute, and %) for each: ice thickness and water depth. This eases evaluation of the performance.

Done.

- RMSD values could be given in units of e.g. 10^-3 to save space and ease reading

Done.
Reflective properties of melt ponds on sea ice

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Abstract. Melt ponds occupy a large part of the Arctic sea ice in summer and affect strongly the radiative budget of the atmosphere-ice-ocean system. In this study, the melt pond reflectance is considered in the framework of the radiative transfer theory. The melt pond is modeled as a plane-parallel layer of pure water upon a layer of sea ice (pond bottom). The reflectance is considered as comprising of Fresnel reflectance by the water surface and multiple reflections between the pond surface and its bottom, which is assumed to be Lambertian. Analytical formulae are put forward to calculate the bidirectional reflectance factor (BRF) and the albedo at different incident angles. The effects of the incident conditions and the atmosphere state are examined. The optical model developed is verified with data from in situ measurements made during several field campaigns performed on landfast and pack ice in the Arctic. The comparison to field spectra demonstrates good performance of the model for the variety of melt pond types observed in the Arctic.

1 Introduction

Melt ponds occupy a large fraction of the Arctic sea ice surface in summer (Barry, 1986; Perovich et al., 2009; Nicolas et al., 2010; Polashenski et al., 2012). They reduce the ice albedo significantly and, therefore, increase the flux of absorbed net light energy and speed up the process of melting, thus amplifying the positive ice albedo feedback effect (Curry et al., 1995; Eicken et al., 2004; Pirazzini, 2008; Schröder et al., 2010). Including light reflection by melt ponds into climate models is an important task. (Floeo et al., 2010; Floeo et al., 2012; Hanke et al., 2013; Lüpkes et al., 2013), particularly in times of the strong environmental changes we see nowadays (Serreze et al., 2000; Tedd et al., 2006; Perovich et al., 2008; Pottier et al., 2014). A physical model of the reflective properties of melt ponds is needed for understanding the physics of sea ice, as well as for the correct interpretation of the results of remote sensing and field measurements (Erikstad et al., 2006; Tschudi et al., 2008; Rösel et al., 2012; Warren, 2013; Zega et al., 2015).

Melt ponds on summer sea ice are also the most variable albedo-affecting factor: they can change from light blue ponds, when just formed, to dark turquoise ones (Perovich, 1996; Barry, 1996; Spreen et al., 2006; Polashenski et al., 2012). Although there are quite a lot of measurements of melt pond spectral albedo (e.g., Perovich, 1994; Makshtas and Lefèvre, 1996; Perovich, 2002, 2009), an adequate physical and optical model of melt pond reflectance is still absent.
and Podgorny (1996) gave the analytical formula expressing the pond albedo in terms of the albedo of its bottom. However, despite asserting that bottom albedo is the main factor that determines the albedo of a pond as a whole, they did not address how to calculate it. This essential gap exists up to now. In this work, we propose the simple solution for the pond bottom spectral albedo. This solution has required the detailed consideration of the inherent optical properties of ice, which forms the pond bottom. Besides, Makshtas and Podgorny (1996) give a formula for pond spectral albedo at direct incidence only; they do not consider the angular distribution of the reflected light. However, just the bi-directional reflectance is measured by satellite optical sensors. Besides, processing of the reflectance measurement data, both satellite and ground-based, requires the atmospheric correction, especially for polar regions. All these points are discussed in this work.

The paper is arranged as follows: Our model of melt pond reflectance is described in Section 2. Subsection 2.1 presents the derivation of the formulas for pond reflectance, given by Makshtas and Podgorny (1996), expanded to various incident conditions. Inherent optical properties (IOPs) of sea ice are considered in subsection 2.2. Simple analytical solution for bottom albedo in terms of the ice IOPs and its thickness is given in subsection 2.3. Subsection 2.4 gives a final brief outline of the developed model. Accounting for the illumination conditions in processing and interpretation of the experimental results are considered in Section 3. The atmospheric correction of experimental data is considered in subsection 3.1. A possibility to use the near IR reflectance as a marker of the ice grains presence is discussed in subsection 3.2. Notes about processing experimental data with common deficient information about incidence are given in subsection 3.3. Then, Section 4 presents the verification of the developed model with the three datasets of in situ measurements (Polarstern-2012, Barrow-2008, and SHEBA-1998). The conclusion sums up the paper.

In this work, we propose a simple optical model that enables the parameterization of the pond bottom albedo with a few physical characteristics and thus determines the spectral reflective properties of the melt pond as a whole, including its bi-directional reflectance.

2 Model description

2.1 Radiance reflected by a melt pond

The model of reflection by melt ponds given in Makshtas and Podgorny (1996) uses the following assumptions:

1. the water layer is an infinite plane-parallel layer;
2. the melt water is pure, with neither absorbing contaminants nor scatterers;
3. the Rayleigh scattering in water is negligible compared to the water absorption; a ray inside the pond is attenuated according to the exponential law;
4. the pond bottom reflects light by the Lambert law (the reflected radiance is independent of the direction).

The described model is illustrated in Fig. 1. In this subsection we repeat the derivation of Makshtas and Podgorny (1996), expanding it to various illumination and observation conditions.

Let $I_b$ be the incident spectral radiance. Then the light intensity (radiance) at the upper pond boundary is $I_{b,n}$.
\[ I_0 - \delta(\mu - \mu_i)N(\phi)E, \]  
(1)

where \( \delta(x) \) is the Dirac delta function, \( \mu = \cos \theta \) (\( \mu_i = \cos \theta_i \)), \( \theta \) is the observation zenith angle (\( \theta_i \) is the solar zenith angle), \( \phi \) is the observation azimuth (counted from the solar principal plane).

The intensity of light reflected from the surface is:
\[ I_r^0 = R' \left( \mu_i, \phi \right) \left( \mu - \mu_i \right) N(\theta) E, \]  
(2)

where \( R' \left( \mu_i, \phi \right) \) is the Fresnel reflectance for incidence angle \( \theta_i \).

When the ray of intensity \( I_0 \) at incidence angle \( \theta_i \) is reflected by the pond boundary, the angle of reflection \( \theta_r^0 \) is given by Snell's law:
\[ n' \sin \theta_i = n \sin \theta_r, \]  
(3)

and the intensity of light just under the boundary is given by the law of conservation of basic radiance:
\[ I_r^1 = I_r^0, \]  
(4)

where \( I_r^1 \) is the Fresnel transmittance for incidence angle \( \theta_i \).

The direct light inside the pond can be divided into the up- and down-welling fluxes. The up-welling flux is the light reflected by the pond bottom. The intensity of the down-welling light inside the pond \( I_d^0 \) consists of the direct light intensity \( I_d^0 \) (the light that was subjected to reflections between the bottom W1 and the surface AW).
\[ I^d = I_d^0 + I_d^0. \]  
(5)

The direct light flux incident on the pond bottom is:
\[ I_d^0 = I_r^0 \delta(\mu - \mu_i)N(\phi)E, \]  
(6)

where \( \delta(\cdot) \) is the extinction coefficient of water, equal to the sum of the water absorption \( \sigma_a \) and scattering \( \sigma_s \) coefficients:
\[ \delta = \sigma_a + \sigma_s. \]  
(7)

We use the data of Segelstein (1981) for the water absorption and the power law for the spectral scattering coefficient:
\[ \sigma_s(\lambda) = \sigma_s \left( \frac{\lambda}{550} \right)^{3} \]  
(8)

where \( \lambda \) is the wavelength and \( \sigma_s = 1.7 \times 10^{-9} \text{ m}^{-1} \); \( \lambda_w = 550 \text{nm} \) (Kopelevich, 1983).

In this work we propose a simple optical model that enables the parameterization of the pond bottom albedo with a few physical characteristics and thus determines the spectral reflective properties of the melt pond as a whole, including its bidirectional reflectance.

2 Model description

2.1 Radiance reflected by a melt pond

The model of reflectance by melt ponds given in Malahoff and Podgorny (1996) uses the following assumptions:

1. the water layer is an infinite plane-parallel layer;
2. the melt water is pure, with neither absorbing contamination nor scattering;
3. the Rayleigh scattering in water is negligible compared to the solar absorption; a ray inside the pond is attenuated according to the exponential law;
4. the pond bottom reflects light by the Lambert law (the reflected radiance is independent of the direction).

The model described in Fig. 1. In this subsection we repeat the derivation of Malahoff and Podgorny (1996), expanding it to various illumination and observation conditions.

Let \( I_0 \) be the incident spectral radiance. Then the light intensity (radiance) at the upper pond boundary is:
\[ I_0 = \delta(\mu - \mu_i)N(\phi)E, \]  
(1)

where \( \delta(x) \) is the Dirac delta function, \( \mu = \cos \theta \) (\( \mu_i = \cos \theta_i \)), \( \theta \) is the observation zenith angle (\( \theta_i \) is the solar zenith angle), \( \phi \) is the observation azimuth (counted from the solar principal plane).

The transmittance of light reflected from the surface is:
\[ I_r^1 = R' \left( \mu_i, \phi \right) \left( \mu - \mu_i \right) N(\theta) E, \]  
(2)

where \( R' \left( \mu_i, \phi \right) \) is the Fresnel reflectance for incidence angle \( \theta_i \).

When the ray of intensity \( I_0 \) at incidence angle \( \theta_i \) is reflected by the pond boundary, the angle of reflection \( \theta_r^0 \) is given by Snell's law:
\[ n' \sin \theta_i = n \sin \theta_r, \]  
(3)

and the intensity of light just under the boundary is given by the law of conservation of basic radiance:
\[ I_r^1 = I_r^0, \]  
(4)

where \( I_r^0 \) is the Fresnel transmittance for incidence angle \( \theta_i \).
The intensity of up-welling light just below the AW interface is:

\[ I_{\text{up}}^o(\mu^*, 0) = I_{\text{up}}(\mu^*, z) \exp \left(-\frac{\beta z}{\mu^*} \right). \] (9)

After the internal reflection the intensity of the down-welling light just below the AW interface is:

\[ I_{\text{down}}^o(\mu^*, 0) = R_e(\mu^*) I_{\text{up}}^o(\mu^*, 0) - R_r(\mu^*) I_{\text{down}}^o(\mu^*, z) \exp \left(-\frac{\beta z}{\mu^*} \right). \] (10)

Where \( R_e \) is the internal reflection coefficient.

The intensity of the down-welling diffuse light at the pond bottom is:

\[ I_{\text{down}}^d(\mu^*, z) = F_d(\mu^*, \nu) I_{\text{down}}^o(\mu^*, 0) \exp \left(-\frac{\beta z}{\mu^*} \right). \] (11)

As the bottom is Lambertian, the intensity of the up-welling light just above the bottom is independent of direction:

\[ I_{\text{up}}^o(\mu^*, z) = I_{\text{up}}^o(\mu^*, z) = I_{\text{up}}^o(z). \] (12)

The boundary conditions at the pond bottom are:

\[ \kappa \Delta m'(z) = K \gamma_0 \kappa_0(\mu, z) I_{\text{up}}^o(z) + \Delta m' \kappa \Delta m', \] (13)

where \( \kappa_0 \) is the bottom albedo and

\[ \kappa_0(z) = \frac{\kappa_0(\mu, \nu) \exp \left(-\frac{\beta z}{\mu^*} \right)}{\Delta m'(z)} \] (14)

From Eq. (1.3) we have:

\[ I_{\text{up}}^o(z) = \frac{\Delta m'}{\kappa(1 - \Delta m'(z))}. \] (15)

Putting together Eqs. (14)- (15), we get for the intensity of light \( I_{\text{up}}^o(z) \) that goes out from the pond:

\[ I_{\text{up}}^o(z) = \frac{\kappa(\mu, \nu) \exp \left(-\frac{\beta z}{\mu^*} \right) \Delta m'(z)}{\kappa(1 - \Delta m'(z))} \] (16)

The total intensity of light reflected by the pond surface is:

\[ I_{\text{down}}^o(\mu, \nu) = \kappa(\mu, \nu) I_{\text{up}}^o(z) \exp \left(-\frac{\beta z}{\mu^*} \right) \] (17)

The bidirectional reflectance factor (BRF) by definition is equal to:

\[ R = \frac{\upsilon^*}{\nu}, \] (18)

Hence the BRF of a melt pond is:

\[ R = \frac{\upsilon^*}{\nu}. \]
The first term describes the sun glint from the AW surface; the second one describes the light multiply reflected between the pond bottom and the surface AW.

The albedo at direct incidence \( A(\mu) \) (the black-sky albedo): 

\[
A(\mu) = \frac{1}{\pi} \int_0^{2\pi} R(\mu, \phi) \, d\phi \, d\theta. 
\]

is found by means of integrating Eq. (19):

\[
A(\mu) = \frac{1}{\pi} \int_0^{2\pi} \frac{\mu}{\mu_{\text{ref}}} \, d\phi \, d\theta. 
\]

where

\[
\mu_{\text{ref}}(\phi) = \frac{\mu}{\mu_{\text{ref}}} \left[ \frac{\xi^2}{\mu^2} \right]. 
\]

10. The albedo at diffuse incidence \( A^o \) (the white-sky albedo):

\[
A^o = \frac{2}{3} A(\mu) - \mu_0 \mu_0. 
\]

is found by integrating Eq. (12):

\[
A^o = A^o + \int_{-\infty}^{\infty} \frac{f_{\text{diff}}(\xi) \, d\xi}{A(\mu, \phi)} 
\]

where \( \mu^{\text{ref}} \) is the Fresnel reflectance for the diffuse incidence.

15. Functions \( f_{\text{diff}}(\xi) \) and \( f_{\text{spec}}(\xi) \) are related by the equation:

\[
f_{\text{spec}}(\xi) = \frac{A(\mu, \phi)}{A(\mu, \phi)} f_{\text{diff}}(\xi),
\]

where \( E_0(\xi) \) is the integral exponential function of the third power:

\[
E_0(\xi) = \int_0^{\xi} \frac{d\xi}{\xi^3}. 
\]

However, the relationship (25) is not very useful in numerical calculations, because these functions are evaluated at different argument values (\( \xi_1 \) and \( \xi_2 \)).

\[
\mu_0 \left( \frac{1}{\mu_0} - 1 \right) = \frac{2}{\pi} \int_0^{2\pi} R(\mu, \phi) \, d\phi \, d\theta. 
\]

From Eq. (13) we have:

\[
I^*(\phi) = \frac{4 A(\mu)}{\pi} \frac{d\phi}{1 - A(\mu, \phi)} 
\]

Putting together Eqs. (46) and (55), we get for the intensity of light \( I^* \) that goes out from the pond:

\[
I^*(\phi) = \frac{4 A(\mu)}{\pi} \frac{d\phi}{1 - A(\mu, \phi)} 
\]

The total intensity of light reflected by the pond is:

\[
I^* = I^* + I^* = I^* (\mu) \left[ \pi - A(\mu, \phi) \right] \frac{d\phi}{1 - A(\mu, \phi)} 
\]

The bidirectional reflectance factor (BRF) by definition is equal to:

\[
R = \frac{I^*}{I^*} 
\]

10. Hence the BRF of a melt pond is:

\[
R = \frac{\pi}{\pi} \frac{4 A(\mu)}{\pi} \frac{d\phi}{1 - A(\mu, \phi)} 
\]

The first term describes the sun specular reflection from the AW surface; the second one describes the light multiply reflected between the pond bottom and the surface AW.

The albedo at direct incidence \( A(\mu) \) (the black-sky albedo):

\[
A(\mu) = \frac{1}{\pi} \int_0^{2\pi} R(\mu, \phi) \, d\phi \, d\theta. 
\]

is found by means of integrating Eq. (19):

\[
A(\mu) = \frac{1}{\pi} \int_0^{2\pi} \frac{\mu}{\mu_{\text{ref}}} \, d\phi \, d\theta. 
\]

where

\[
\mu_{\text{ref}}(\phi) = \frac{\mu}{\mu_{\text{ref}}} \left[ \frac{\xi^2}{\mu^2} \right]. 
\]
2.1 Inherent optical properties of substrate ice

The main factor in Eqs. (19), (21), and (24) that determines the melt pond reflection is its bottom albedo \( \alpha_b \). In order to calculate it we should first consider the inherent optical properties (IOPs) of substrate ice that forms the pond bottom.

The IOPs of a medium used in the radiative transfer theory are the spectral scattering \( \sigma(\lambda) \) and absorption \( \alpha(\lambda) \) coefficients and the scattering phase function \( p(\theta) \). In the following consideration, as in other numerous radiative transfer theory applications, the transport scattering coefficient \( \sigma_t \) is used:

\[
\sigma_t = \sigma(1 - g),
\]

where \( g \) is the average cosine of the scattering angle \( \theta \):

\[
g = \frac{1}{2} \int_0^{2\pi} p(\theta) \cos \theta \sin \theta \, d\theta.
\]

The transport coefficient is useful in calculating the reflection and transmission by a scattering layer with very elongated phase function, particularly if one is interested in the layer albedo, rather than the angular structure (BRDF) of the reflected light (Zoge et al., 1991).

Main factors that determine optical properties of sea ice are its microphysical structure and values of complex refractive indices of its constituents; the dispersion of complex refractive indices determines the spectral properties of ice.

As the volume concentration of air bubbles in sea ice is small — only up to \( \approx 5\% \) even in the extremely bubbly ice (Gavrilov and Gorbushkii, 1986) — and the complex refractive index of brine is very close to that of ice (see Baturin et al., 1994; Warren and Brandt, 2009; and Sec. 2.2b), we take the absorption coefficient of sea ice equal to that of solid ice. Impurities — sediment and organic materials from snow water — could change absorption coefficients, particularly at shorter wavelengths. At this stage we neglect their effect, keeping in mind that their absorption spectra can be easily added, if necessary.

The scattering takes place at inhomogeneities in sea ice and is mainly caused by air bubbles and brine inclusions (Mobley et al., 1998; Light, 2010). Another source of scattering could be salt crystals, but they precipitate at low temperatures and are not observed in summer ice, where melt ponds are formed. Minerals (NaCl, NaNO\(_3\)) begin to precipitate at \(-8^\circ \text{C}\) and 
hydrates (NaCl NH\(_2\)O) at about \(-27^\circ \text{C}\) (Light et al., 2003).

a) Air bubbles

The upper layer of sea ice (20-30 cm thick) contains usually significant amounts of air bubbles (Gavrilov and Gorbushkii, 1986; Mobley et al., 1998), with volume concentration, which can reach values of \( \approx 5\% \) and decreases with depth. (We do not consider here the highly scattering surface layer that forms in the melting process and is commonly referred to as “white ice”). Air bubbles in sea ice are mostly spherical. Light (2010) gives the following size distribution for bubbles in the first year ice:

\[
is found by integrating Eq. (24):

\[
X^b = X^0 + \frac{\int_{\lambda} \int \sigma^b(\lambda, \theta)}{\int \sigma^b(\lambda, \theta) \, d\lambda}
\]

where \( X^b \) is the total reflectance for the difference incidence.

Functions \( f_r(\lambda) \) and \( f_s(\lambda) \) are related by the equation

\[
f_r(\lambda) = f_s(\lambda)\left(1 - \lambda(\lambda)\right),
\]

where \( E(\lambda) \) is the integral exponential function of the third power:

\[
E(\lambda) = \frac{1}{\lambda^3}.
\]

However, the relationship (25) is not very useful in numerical calculations, because these functions are evaluated at different argument values (\( \intn \) and \( 2\pi \)). The way to speed up simulations is to calculate these functions once for a given set of wavelengths and then to use a look-up table. All the other formulas given in the manuscript are completely analytical and can be used straightforward.

2.2 Inherent optical properties of under-ice pond

The main factor in Eqs. (19), (21), and (24) that determines the melt pond reflection is its bottom albedo \( \alpha_b \). In order to calculate it we should first consider the inherent optical properties (IOPs) of under-ice pond that forms the pond bottom.

The IOPs of a medium used in the radiative transfer theory are the spectral scattering \( \sigma(\lambda) \) and absorption \( \alpha(\lambda) \) coefficients and the scattering phase function \( p(\theta) \). In the following consideration, as in other radiative transfer theory applications (see, e.g., Davidson, 1958; Chandrasekhar, 1960), the transport scattering coefficient \( \sigma_t \) is used:

\[
\sigma_t = \sigma(1 - g),
\]

where \( g \) is the average cosine of the scattering angle \( \theta \):

\[
g = \frac{1}{2} \int_0^{2\pi} p(\theta) \cos \theta \sin \theta \, d\theta.
\]

The transport coefficient is useful in calculating the reflection and transmission by a scattering layer with a very forward peaked phase function, particularly if one is interested in the layer albedo, rather than the angular structure (BRDF) of the reflected light (Zoge et al., 1991).

Main factors that determine optical properties of sea ice are its microphysical structure and values of complex refractive indices of its constituents; the dispersion of complex refractive indices determines the spectral properties of sea ice.
Gavrilov and Gaikushko (1978) report the presence of much larger bubbles in the bubbly ice (from 0.1 to 2 mm with the exponent 1.24).

However, since air bubbles in ice are optically hard (the refractive index of air differs strongly from that of ice) and do not absorb light, scattering by bubbles of this size range is described by the laws of geometrical optics. Thus, the scattering characteristics do not depend on the bubble size (unless considering the strictly forward and backward directions), the shape of the size distribution is also insignificant. Particularly, the scattering efficiency $Q_{sc}$ in this size range equals 2 and the phase function can be easily calculated with the Mie formulations for any type of size distribution, e.g., for the one given in Eq. (29).

The refractive index of air (relative to ice) in the interval 0.35-0.95 μm changes from 0.755 to 0.768 with average value of 0.763 within this interval. The corresponding average cosine $g$ obtained with the Mie calculations, takes values from 0.851 to 0.865 with the mean value of 0.860, and therefore the spectral variability does not exceed 2%.

b) Brine inclusions

The main features of brine inclusions are the following: they are optically soft, i.e., their refractive index $n_b$ (brine relative to ice) is close to unity; their size is large (compared to the wavelength), and their shape is strongly irregular. Applying these features we can apply the approximation for irregularly shaped particles developed by Malinka (2015) to describe scattering properties of brine inclusions.

The size of brine inclusions, which can be of the order of several millimeters, is so much larger than the wavelengths of visible light that their optical properties can be considered in the limit of infinitely large particles, despite their refractive index $n_b$ is close to unity:

$$n_b - 1 \ll 1.$$  

In this limit the size distribution is also insignificant and the scattering efficiency $Q_{sc}$ is independent of the wavelength:

$$Q_{sc} = 2.$$  

The scattering phase function of optically soft particles can be approximated according to Malinka (2015) by:

$$p(\theta) = \frac{2\pi^2 (1 + \mu^2)}{(1 + 2\mu^2 \cos^2 \theta)}.$$  

where $\mu = \cos \theta$ and $x$ is the optical particle size (dimensionless), which for large particles equals:

$$x = n_b \frac{4\pi r}{\lambda}.$$  

In view of Eqs. (30) and (31), we can write:

$$N(\mu) \approx \mu^{-1}, \quad 4\mu \leq r \leq 70 \mu m.$$  

Gavrilov and Gaikushko (1978) report the presence of much larger bubbles in the bubbly ice (from 0.1 to 2 mm with the exponent 1.24).

However, since air bubbles in ice are optically hard (the refractive index of air differs strongly from that of ice) and do not absorb light, scattering by bubbles of this size range is described by the laws of geometrical optics. Thus, the scattering characteristics do not depend on the bubble size (unless considering the strictly forward and backward directions), and the shape of the size distribution is also insignificant. Particularly, the scattering efficiency $Q_{sc}$ in this size range equals 2 and the phase function can be easily calculated with the Mie formulations for any type of size distribution, e.g., for the one given in Eq. (29).

The refractive index of air (relative to ice) in the interval 0.35-0.95 μm changes from 0.755 to 0.768 with average value of 0.763 within this interval. The corresponding average cosine $g$ of the scattering angle, obtained with the Mie calculations, takes values from 0.851 to 0.865 with the mean value of 0.860, and therefore the spectral variability does not exceed 2%.

b) Brine inclusions

The main features of brine inclusions are the following: they are optically soft, i.e., their refractive index $n_b$ (brine relative to ice) is close to unity; their size is large (compared to the wavelength); and their shape is strongly irregular.
The average cosine $g$ of the phase function (32) is:

$$g = \frac{1}{\kappa^2 - 1}.$$  

(34)

Figure 2 demonstrates the spectral dependence of the refractive index of water relative to ice. We used the data by Warren and Brandt (2008) for ice. The refractive index of water is taken from Hall and Querry (1975), Segelstein (1981), Daunon and Masunara (2007), and Koczenburg et al. (2012) for distilled water, formula of Quan and Fry (1998) for brine of zero salinity at temperature $0^\circ$C, and formula of Finrved (2009), which is based mainly on Quan and Fry (1995) and the measurements by Maykut and Light (1995), for brine with equilibrium salinity at temperatures $-2^\circ$C, $-4^\circ$C, and $-6^\circ$C. The earlier data (Hall and Querry, 1973 and Segelstein, 1981) clearly demonstrated the spectral dependences, their dispersions being opposite. In contrast, the newer data do not demonstrate such dependence; according to the more modern data, the refractive index (relative to ice) of water, including brine, is almost spectrally neutral. This question is important when describing the light scattering by brine inclusions in ice, because the transport scattering coefficient is determined mainly by the value $(\eta_1 - 1)^2$ (see Eqs. (27) and (34)-(35)). Finally, according to the newer data we will accept that the relative refractive index of brine, and therefore the transport scattering coefficient of brine inclusions, is spectrally neutral.

For example, $\eta = 1.024$ for temperature $-2^\circ$C and, according to Eqs. (34)-(35), $\frac{\eta_1}{\eta_0} = 0.999$. Note that the value of $g_0$ in this model is significantly greater than that used in many other studies, e.g., in Mosesley et al. (1998) or Light et al. (1981).

### Inherent optical properties of sea ice

Light scattering properties of sea ice are a combination of those of brine inclusions and air bubbles. The total and transport scattering coefficients are the sum of the respective values:

$$\sigma = \sigma_b + \sigma_a,$$

$$\sigma_t = \sigma_b + \sigma_a.$$  

(36)

We denote the values related to brine inclusions with the subscript $b$ and to air bubbles with the subscript $a$. The phase function and the average cosine are the linear combination of the respective values:

$$p(\theta) = \frac{\sigma_b}{\sigma} p_b(\theta) + \frac{\sigma_a}{\sigma} p_a(\theta),$$

$$1 - g = \frac{\sigma_a}{\sigma} (1 - g_a) + \frac{\sigma_b}{\sigma} (1 - g_b).$$  

(37)

Once $g_a$ and $g_b$ are known (e.g., at $-2^\circ$C $g_a = 0.86$, $g_b = 0.998$), the resulting $g$ depends only on the proportion of fractions $a$ and $b$.

$$...$$  

(55)
Generally, the 10PS of sea ice depend on its microstructure. In view of the fact that both bubble and brine inclusion size is much larger than the wavelength, the scattering coefficient equals:

$$\sigma_j = 2\pi \nu_j \quad (j = a, b), \quad (38)$$

where $\nu_j$ is the specific cross-sectional area of inclusions (air or brine):

$$\nu_j = \left[ \frac{S_j}{N_j} \right] \frac{\lambda^2}{4\pi}, \quad (39)$$

Here subscript $j$ shows the fraction number, $\left[ \frac{S_j}{N_j} \right]$ is the average cross-sectional area of $j$-inclusions, $R_j$ their effective radius, $N_j$ and $C_j$ are their numeric and volume concentration, respectively.

The phase function (and consequently its average cosine $g$) can be characterized by the ratio of volume concentration air-to-brine $C_a/C_b$, if their effective radii are determined. E.g., for bubbles size distribution (29), the effective radius is $R_b = 42.55 \mu m$. Light (2010) gives the value of 110 m$^{-1}$ for $\nu_a$, the specific cross-sectional area of brine inclusions, for a sample of typical first year ice at $-15^\circ C$, which can grow up to 400 m$^{-1}$ when warming. The estimate, made by Light (2010) for the brine volume concentration in the same sample, gives the values from 1.2% to 1.9%. This allows us to estimate the effective radius of brine inclusions as $X_b = 100 \mu m$.

However, as the morphotropy of sea ice can vary drastically with place and time, the more convenient way to characterize the ratio of air and brine fractions is to use the ratio of their transport coefficients $\sigma_a/\sigma_b$. This ratio is related to the ratio of volume concentrations as:

$$\frac{\sigma_a}{\sigma_b} = \frac{1 - \bar{g}_b R_b C_b}{1 - \bar{g}_a R_a C_a}, \quad (40)$$

Figure 3 presents the phase function of mixtures with different air-to-brine fractions ratio.

We conclude that the phase function (and consequently $g$) of sea ice is spectrally neutral in the visible and near IR range. In virtue of Eq. (38), the scattering coefficient $\sigma$ also spectrally neutral. Consequently, the transport scattering coefficient $\sigma_t$ is also spectrally neutral and can serve as a scalar parameter that characterizes scattering in sea ice.

2.3 Bottom albedo

If both the absorption and transport scattering coefficients are known, the albedo of a layer can be calculated within the two-stream approximation, which is widely used for practical calculations:

$$A = A_a + \frac{1 - \exp(-2\pi)}{1 - 2\pi \exp(-2\pi)}, \quad (41)$$

will accept that the relative reflective index of brine, and therefore the transport scattering coefficient of brine inclusions, is spectrally neutral. E.g., $\bar{g}_b = 1.02$ for temperature $-2^\circ C$ and, according to Eq. (34), $g_b = 0.954$. Note that the value of $g$ in this model is significantly greater than that used in many other studies, e.g., in Mulvey et al. (1993) or Light et al. (1998).

\[\text{c) Inherent optical properties of sea ice}\]

Light scattering properties of sea ice are a combination of those of brine inclusions and air bubbles. The total and transport scattering coefficients are the sum of the respective values:

$$\sigma = \sigma_a + \sigma_b, \quad \sigma_t = \sigma_{a} + \sigma_{b} \quad (56)$$

We denote the values related to brine inclusions with the subscript $b$ and to air bubbles with the subscript $a$. The phase function and the average cosine are the linear combination of the respective values:

$$\bar{g}(\theta) = \frac{\sigma_b}{\sigma} \bar{g}_b(\theta) + \frac{\sigma_a}{\sigma} \bar{g}_a(\theta), \quad (57)$$

Once $\sigma_a$ and $\sigma_b$ are known (e.g., at $-2^\circ C$, $\sigma_a = 0.88$, $\sigma_b = 0.998$), the remaining $g$ depends only on the proportion of fractions $\sigma_a$ and $\sigma_b$.

Generally, the 10PS of sea ice depend on its microstructure. In view of the fact that both bubble and brine inclusion size is much larger than the wavelength, the scattering coefficient equals:

$$\sigma_j = 2\pi \nu_j \quad (j = a, b), \quad (38)$$

where $\nu_j$ is the specific cross-sectional area of inclusions (air or brine):

$$\nu_j = \left[ \frac{S_j}{N_j} \right] \frac{\lambda^2}{4\pi}, \quad (39)$$

Here subscript $j$ shows the fraction number, $\left[ \frac{S_j}{N_j} \right]$ is the average cross-sectional area of $j$-inclusions, $R_j$ their effective radius, $N_j$ and $C_j$ are their numeric and volume concentration, respectively.

The phase function (and consequently its average cosine $g$) can be characterized by the ratio of volume concentration air-to-brine $C_a/C_b$, if their effective radii are determined. E.g., for bubbles size distribution (29), the effective radius is $R_b = 42.55 \mu m$. Light (2010) gives the value of 110 m$^{-1}$ for $\nu_a$, the specific cross-sectional area of brine inclusions, for a sample of typical first year ice at $-15^\circ C$, which can grow up to 400 m$^{-1}$ when warming. The estimate, made by Light (2010) for the brine volume concentration in the same sample, gives the values from 1.2% to 1.9%. This allows us to estimate the effective radius of brine inclusions as $X_b = 100 \mu m$.

However, as the morphotropy of sea ice can vary drastically with place and time, the more convenient way to characterize the ratio of air and brine fractions is to use the ratio of their transport coefficients $\sigma_a/\sigma_b$. This ratio is related to the ratio of volume concentrations as:

$$\frac{\sigma_a}{\sigma_b} = \frac{1 - \bar{g}_b R_b C_b}{1 - \bar{g}_a R_a C_a}, \quad (40)$$

Figure 3 presents the phase function of mixtures with different air-to-brine fractions ratio.

We conclude that the phase function (and consequently $g$) of sea ice is spectrally neutral in the visible and near IR range. In virtue of Eq. (38), the scattering coefficient $\sigma$ also spectrally neutral. Consequently, the transport scattering coefficient $\sigma_t$ is also spectrally neutral and can serve as a scalar parameter that characterizes scattering in sea ice.
where $A_0$ is the albedo of the semi-infinite layer with the same optical characteristics, $\gamma$ is the asymptotic attenuation coefficient, and $\tau$ is the layer optical thickness. The version of the two-stream approximation developed by Zege et al. (1991) expresses these characteristics as follows:

$$A_0 = 1 - \left(\frac{\sigma_0 \gamma_0}{\gamma (\tau + 2)}\right),$$

$$\tau = \frac{3}{4 \sigma_0 + \gamma_0} \sqrt{(\tau + 2)},$$

$$z = \frac{\sigma_0}{\sigma_0 + \gamma_0} \gamma,$$  \hspace{1cm} (42)

where $\sigma_0$ is the ice absorption coefficient; $\gamma$ is the ice layer thickness.

The two-stream approximation in the version given in Zege et al. (1991) has a wide range of applicability and can be used both for strongly and weakly absorbing media, for optically thin and thick layers. Hence, this approximation can be applied to all the variety of melt ponds, from young ponds, which are light blue and have comparatively optically thick ice substrate, to mature dark ones, where substances are optically thin.

### 2.4 Model outline

Thus, in the assumption of a Lambertian bottom and plane parallel geometry, which assumes the absence of strong wind, i.e., calm pond surface, the spectral reflection of ponds is determined by two values: water layer depth $\tau$ and the albedo of the pond bottom $A_0$. The latter, in turn, depends on the transport scattering coefficient of sub-surface ice $\sigma_0$ and its geometric thickness $\gamma_0$, respectively. The transport optical thickness $\sigma_0 \gamma_0$ and $\gamma_0$ have a special behavior, whereas the others $\sigma_0$, $\gamma_0$ and $\gamma$ are scalar.

The outlined model of a melt pond is shown in short in Table 1.

### 3 Illumination conditions

#### 3.1 Atmospheric correction

Correct processing of the reflectance measurement results requires the correct modeling of the illumination conditions. This is especially important for measurements in the Arctic, because of the low sun and the bright surface. When the sky is overcast, the incident light is close to diffuse, even if the solar disk is visually observed (Malinka et al., 2016b). In this case, the measured albedo is the white-sky one. However, when the sky is clear and the sun is near the horizon, the direct solar flux is comparable to the diffuse flux from the sky, so the measured (blue-sky) albedo value is a mixture of those at direct (black-sky) and diffuse (white-sky) incidence. The black-sky albedo increases when the sun is approaching the horizon, so the
difference between the white- and black-sky albedos is most essential at oblique incidence (see Fig. 4). The problem of the correct interpretation of the measured blue-sky albedo is considered in detail in Malinka et al. (2016b) for a homogeneous surface. However, the albedo of a melt pond can differ significantly from that of the surrounding background, e.g., white ice or snow. Some estimation for this case is given below.

Let $R$, $A_0$, and $A'$ be, as before, the BRF, black-sky, and white-sky albedo of a melt pond, respectively. Let the surrounding background be Lambertian with albedo $\tau_0$. Then the brightness of the incident radiance can be estimated as (Malinka et al., 2016b):

$$ B_i = \frac{1}{\tau_0} \left( \frac{t_0}{1 - \tau_0} - T_0 \right) \frac{E_0}{\pi}, \quad (44) $$

where $t_0(\mu)$ and $t_{\mu}(\mu)$ are the direct and diffuse atmosphere transmittances, $\delta = \sin(\mu - \mu_0) \sin(\theta - \theta_0) / \mu_0$ is the identity operator ($\delta(\sigma)$ is the Dirac delta-function), $T(\mu) = t_{\mu}(\mu) + t_0(\mu)$ is the atmosphere transmittance at direct incidence, and $\theta_0$ is the atmospheric bidirectional reflectance at incidence from below. $E_0$ is the extraterrestrial solar irradiance.

So, the light flux incident to a melt pond is:

$$ F_i = \frac{R_i(\mu) E_0}{1 - \tau_0}. \quad (45) $$

The radiance of light reflected by pond follows from Eq. (44):

$$ B_i = \left( R_0(\mu - \mu_0) - A_0 \right) \left( \frac{t_0}{1 - \tau_0} - T_0 \right) \frac{E_0}{\pi} $$

$$ \left( B(\mu, \mu_0, \mu_0) - A(\mu) \right) t_0(\mu) \frac{E_0}{\pi} \quad (46) $$

Therefore the reflected flux is:

$$ F_r = \left( A(\mu) - A_0 \right) \left( \frac{t_0}{1 - \tau_0} - T_0 \right) E_0 \tau_0. \quad (47) $$

For the measured value of the blue-sky albedo $A_0$ it follows:

$$ A = \frac{A_0 \left( 1 - A_0 \right)}{1 - \tau_0}. \quad (48) $$

The equation for the blue-sky albedo can be written as a linear combination of the black and white-sky albedos:

$$ \alpha = \alpha_0 R_0(\mu_0) + (1 - \alpha_0) A_0(\mu). \quad (49) $$

with the proportion of direct radiance $\alpha_0$:

$$ \alpha_0 = \frac{t_0(\mu)}{t_0(\mu)} \frac{1}{1 - \tau_0}. \quad (50) $
Factor \(1 - \varepsilon\) is responsible for multiple reflections between the atmosphere and surrounding background. Albedo spectra of a light melt pond (or pond with high reflectance) at different illumination conditions are shown in Fig. 5. The angle of incidence is 80° (the sun elevation is 10°). The interval of albedo changes is limited by the values of white and black-sky ones. Also shown are the blue-sky albedo for clear sky and for sky with thin cirrus layer (with optical thickness of 0.1). Both are considered with different surrounding backgrounds: perfectly black \((\varepsilon_1 = 0)\) and white \((\varepsilon_2 = 1)\). As seen from Fig. 5, the effect of background is negligible, so the results of melt pond albedo measurements can be processed without a priori knowledge of the albedo of surrounding background.

### 3.2 IR reflectance

In contrast to the visible range, ice and water absorb a significant amount of light in the IR; a few centimeters thick layer of ice or water completely absorbs radiation in the infrared range. Thus, the melt pond optical response in the IR is restricted to the Fresnel reflection by the pond surface. In contrast, ice grains in white ice are of the order of millimeters in size (and even smaller in snow). Due to this fact one can trace the appearance of the specific features of the behavior of the imaginary part of the ice refractive index \(k\) in the IR in the reflection spectra of white ice and snow. In particular, \(k\) has a local minimum at 1.1 \(\mu\)m, which provides a slight peak of reflection in the interval 1.09-1.11 \(\mu\)m (Wiscombe and Warren, 1980). Figure 6 shows an example of the albedo spectral dependence for white ice, snow, and a melt pond. It clearly demonstrates that for wavelengths longer than 0.9 \(\mu\)m the melt pond reflection is restricted by the Fresnel reflection as a constant value, while snow and white ice demonstrate a local maximum at 1.1 \(\mu\)m. Thus, this slight peak can be served as a criterion for determining if a spectrum is taken entirely from an open pond or partially from snow/ice surface. If this peak is observed in a measured spectrum, it clearly indicates the presence of ice grains (of white ice or snow) in the receiver field of view.

### 3.3 Measurement geometry

In the description of the field data used in this study snow/ice conditions were reported as overcast. Only a few measurements were taken under clear sky conditions. Scattered clouds were not reported at all in the measurement series considered, likely due to challenges gathering accurate albedo measurements in variable illumination. In the case of overcast sky, the measured albedo was interpreted as the white-sky one. In the clear sky cases, the Rayleigh atmosphere with the Amuri Background aerosol (Tomasi et al., 2007) was assumed. In this case the incident angle was determined from the pond reflection in the IR at the interval 1.25-1.3 \(\mu\)m (prominently) or 0.85-0.9 \(\mu\)m, if data at the former interval are not available. As the IR signal (both incident and reflected) is quite weak and hence some noise is always noticeable, we average the signal over one of the abovementioned intervals. The pond reflectance in these IR intervals is completely determined by the Fresnel reflection of its upper boundary. Atmospheric scattering in the IR is negligible (especially at 1.3 \(\mu\)m), so the incident light is unidirectional. In this situation the incident angle can be easily calculated through the Fresnel formulas.

\[
\begin{align*}
F_\text{IR} &= \frac{\tau_\text{IR}(\mu_1, \varphi)}{1 - x_\text{IR}} \times F_\text{solar}, \\
E_\text{IR} &= \frac{\phi_\text{IR}(\mu_1, \varphi)}{1 - x_\text{IR}} \times E_\text{solar}.
\end{align*}
\]

where \(\tau_\text{IR}(\mu_1, \varphi)\) and \(\phi_\text{IR}(\mu_1, \varphi)\) are the direct and diffuse atmosphere transmittances, \(d = \pi \mu \sigma_\text{IR}(\varphi, \alpha, \mu_1)\) is the identity operator (\(x_\text{IR}\) is the Dufour delta-function), \(\Phi_\text{IR}(\mu_1, \varphi) = \phi_\text{IR}(\mu_1, \varphi) + \phi_\text{IR}(\mu_1, \varphi)\) is the atmospheric transmittance at direct incidence, and \(F_\text{solar}\) is the atmospheric hemispherical reflectance at incidence from below. \(F_\text{solar}\) is the extra-solar thermal irradiance.

So the light flux incident to a melt pond is:

\[
E_\text{pond} = \frac{\tau_\text{IR}(\mu_1, \varphi)}{1 - x_\text{IR}} \times F_\text{solar}.
\]

The radiance of light reflected by pond (from Eq. (44))

\[
E_\text{pond} = \frac{\phi_\text{IR}(\mu_1, \varphi)}{1 - x_\text{IR}} \times F_\text{solar}.
\]

Therefore the reflected flux is:

\[
E_\text{pond} = \left(\frac{d\mu}{d\alpha}\right) \phi_\text{IR}(\mu_1, \varphi) + \frac{\Phi_\text{IR}(\mu_1, \varphi)}{x_\text{IR}} E_\text{solar}.
\]

For the measured value of the blue-sky albedo \(\varepsilon_\text{b}\) it follows
\[
\varepsilon_\text{b} = \frac{\frac{E_\text{pond}}{E_\text{solar}} - \frac{E_\text{pond}}{E_\text{solar}} \left(1 - \varepsilon_\text{w}\right)}{\frac{E_\text{pond}}{E_\text{solar}} \Phi_\text{IR}(\mu_1, \varphi)}.
\]

The equation for the blue-sky albedo can be written as a linear combination of the black and white-sky albedos:
\[
\varepsilon_\text{b} = w \varepsilon_\text{w},
\]

with the proportion of direct irradiance

\[
\varepsilon_\text{w} = \frac{\varepsilon_\text{w}}{1 - \varepsilon_\text{w}}.
\]

Factor \((1 - \varepsilon)\) is responsible for multiple reflections between the atmosphere and surrounding background. Modeled albedo spectra of a light melt pond (or pond with high reflectance) at different illumination conditions are shown in Fig. 5. The angle of incidence is 80° (the sun elevation is 10°). The interval of albedo changes is limited by the values of white and black-sky ones. Also shown are the blue-sky albedo for clear sky and for sky with thin cirrus layer (with optical thickness of 0.1). Both are considered with different surrounding backgrounds: perfectly black \((\varepsilon_1 = 0)\) and white \((\varepsilon_2 = 1)\). As seen from Fig. 5, the effect of background is negligible (only small difference between blue lines and dotted/crossed lines), so the results of melt pond albedo measurements can be processed without a priori knowledge of the albedo of the surrounding background.
4 Model verification

Three different datasets with in-situ field measurements were used for the evaluation of the pond model. They are described in the next subsections.

4.1 Polarcirkel-2012

The measurements of spectral albedo of the Arctic surfaces were carried out during the R/V Polarcirkel cruise ARK-XXVI/3 (August 2 – October 8, 2012). Only in the second half of the cruise did the vessel leave the marginal ice zone and enter the ice pack. The ice thickness varied from 0.5 to 3 meters with an average of 1.1-1.5 meter. The melt ponds observed were both open and frozen over, sometimes snow covered. The data were collected during stationary, when the vessel was parked at an ice floe for several days. This gave the possibility to obtain several-day data sequences of melting sea ice and forming melt ponds at the same location.

The ASD FieldSpecPro III spectroradiometer used for these measurements has three different sensors that provide measurements from 350 nm to 2500 nm with the spectral resolution of 1 nm. A 10 x 10 cm² Spectralon white plate served as a diffuser, which was held at about 1 meter above the surface and was directed first towards the measured surface and then towards the sky. The ratio of the two measurements gives the hemispherical reflectance (albedo) of the surface. For some cases the water depth and ice thickness within the pond were measured.

For the model verification we considered the melt pond albedo in the spectral interval 0.35 – 1.3 μm. The retrieval procedure implies searching for the pond parameters values (see Table 1) that provide the best fit (in the sense of the least squares) of the measured and modeled spectra. For the cases where the pond depth and underlying ice thickness were known the retrieved pond parameters were compared to the measured ones.

Some ponds were frozen over, i.e., they had a layer of newly formed ice on top of their surface. It is evident that a layer of transparent ice at the pond surface practically does not change pond reflectance, so we consider the pond with ice crust as in the same manner as open ones. However, if the upper ice layer is bubbly or snow covered, the pond reflectance can change drastically: the pond gets brighter and may become indistinguishable from the surrounding ice in the visible range. These snow-covered ponds would require other means for their characterization. We exclude such cases from the consideration.

Figures 7-10 present photos of different ponds and their reflectance spectra, measured and simulated with the retrieved parameters (denoted as "retrieved" in the legend).

Figure 7 shows the photos, modeled and measured spectra of light blue melt ponds with uniform bottom on thick first-year ice under clear and cloudy skies, measured in the Central Arctic on 10.08.2012, 10.09.2012 and 22.08.2012, respectively. In all cases the ponds are frozen over with a 2-3 cm layer of ice on top. Figure 8 shows three cases of frozen over blue ponds with heterogeneous bottom over land fast ice measured on 11.08.12, 22.08.12 and 22.09.12, respectively. One can see darker parts in the ponds, which result from ice melting from the lower boundary. Figure 9 presents dark open melt ponds on thinner first-year ice under overcast skies, all measured on 26.08.2012. The albedo of these ponds is much lower than 0.

3.2 IR reflectance

In contrast to the visible range, ice and water absorbs a significant amount of light in the IR: a few centimeter thick layer of ice or water completely absorbs radiation in the infrared range. Thus the melt pond optical response in the IR is restricted to the Fresnel reflection by the pond surface. In contrast, ice grains in white ice are of the order of millimeters in size (and even smaller in snow). Due to this fact, the appearance of specific features in the IR portion of the reflection spectrum can be used to diagnose the inclusion of unexpanded ice and snow. In particular, the imaginary part of the refractive index 2 has a local maximum at 1.1 μm, which provides a slight peak of reflectance in the interval 1.05-1.1 μm (Wiscombe and Warren, 1980).

Figure 6 shows an example of the modeled: albedo’s spectral dependence for white ice, snow, and a melt pond. It clearly demonstrates that for wavelengths longer than 0.6 μm the melt pond reflectance is restricted by the Fresnel reflection to a constant value, while snow and white ice demonstrate a local maximum at 1.1 μm. Thus, this peak can serve as a criterion for determining if a spectrum is taken entirely from an open pond or partially from snow/ice surface. If this peak is observed in a measured spectrum, it clearly indicates the presence of ice grains (of white ice or snow) in the receiver field of view.

3.3 Measurement geometry

In the description of the field data used in this study, most sky conditions were reported as evenning. Only a few measurements were taken under clear sky conditions. Scattered clouds were not reported at all in the measurement series considered. In the cases of evening sky, the measured albedo was interpreted as the white sky one. In the clear sky cases, the Rayleigh atmosphere with the Arctic Background aerosol (Tomasi et al., 2007) was assumed. In this case the solar incidence angle was determined from the pond reflection in the IR: at the interval 1.25-1.3 μm (preferentially at 0.75-0.9 μm, if data at the former interval are not available). As the IR signal (both incident and reflected) is quite weak and hence some noise is always noticeable, we average the signal over one of the abovementioned intervals. The pond reflectance in these IR intervals is completely determined by the Fresnel reflection of its upper boundary. Atmospheric scattering in the IR is negligible (especially at 1.3 μm), so the incident light is unidirectional. In this case the solar incidence angle can be calculated through the Fresnel equations.

4 Model verification

Three different datasets with in-situ field measurements were used for the evaluation of the pond model. They are described in the next subsections.

4.1 Polarcirkel-2012

Measurements of spectral albedo of different sea ice surfaces were carried out during the R/V Polarcirkel cruise ARK-XXVI/3 (August 2 – October 8, 2012). Only in the second half of the cruise did the vessel leave the marginal ice zone and enter the ice pack.
than that of the previous ones: from about 0.07 to 0.14 in the visible and about 0.05 in the IR. Figure 10 presents the two cases of light blue ponds both measured on 26.08.12 and a dark pond contaminated with algae aggregates measured on 21.06.2012, all under overcast skies. Surprisingly, the spectrum of the pond with algae is reproduced quite well. This is because the contribution of the yellow algae spots to a total reflection is proportional to their area, which is not very large. However, their effect can be clearly seen in the spectrum: the measured values are less than the modeled ones in the blue range (0.3-0.5 μm) and greater in the yellow-green (0.5-0.6 μm). The above ponds are quite different: from dark to very light blue in color, open and frozen over, clear and contaminated with organic matter. In spite of this, the model is able to reproduce the measured spectra in the visible region with high accuracy in all studied cases. The root-mean-square difference (RMSD) between the measured and simulated spectra has the average value of 0.01 for the whole considered spectrum and 0.007 for the visible range. The retrieved and measured geometrical parameters of the ponds, as well as the RMSD between the measured and simulated spectra, are presented in Table 2 and shown in Fig. 14.

4.2 Barrow-2008

Melting pond spectra observed in Barrow were collected as part of the SIZONET program observing pond formation (Polashenski et al., 2012). Observations were collected at sites approximately 1 km offshore from Nikolsk on landfast ice in the Chukchi Sea, near 71.366N, 156.524E on several first-year ice. For this work, a total of 27 measured melt pond spectra were used (no photographs were taken). All melt ponds were dark and polluted with sediments and their spectral looks quite similar. Three of them are presented in Fig. 11. The albedo does not exceed the value of 0.3 in its maximum and show a discrepancy in the blue range, presumably due to the presence of mineral sediments. Because of this, the RMSD between the measured and simulated spectra for the visible range (0.01) is greater than that for the whole spectrum (0.009). The ice thickness was not measured. The pond depths, measured and retrieved, as well as the RMSD, are shown in Table 2 and Fig. 14.

4.3 SHEBA-1998

SHEBA was a year-long drift experiment conducted in the Beaufort Sea from October 1997 – 1998 (Perovich et al., 1999; Uttal et al., 2002). Intensive measurements of the properties and processes of the atmosphere-ice-ocean system were made. This included observations of the spatial variability and temporal evolution of the spectral albedo of the ice cover (Perovich et al., 2002).

One pond in this expedition was especially interesting, because its bottom had a region that was much brighter than the surrounding bottom. This region had sharp borders and rectangular corners (see the photo in Fig. 12). This likely was a broken piece of bubbly multiyear ice that was incorporated into the ice cover. This piece of ice had more air bubbles than the darker adjacent ice. This dual pond was observed during the entire period of its formation and development; the spectra were entered into the ice pack. The ice thickness varied from 0.5 to 3 meters with an average of 1.3-1.5 meter. Melt ponds were observed in August. The ponds were both open (with no snow ice) and frozen over (with a thin layer of ice), sometimes snow-covered. The data were collected during storms, when the ice was packed at an ice floe for several days. This gave the possibility to obtain several day data sequences of melting sea ice and evolving melt ponds at the same location. The stations, where ponds were observed, were located from 84°31’N 137°35’E to 82°56’N 129°44’E. For more information about the cruise, see Bierini et al. (2012) and Bierini et al. (2016). The ASD FieldSpecPro III spectroradiometer used for these measurements has three detectors that provide measurements from 350 nm to 2500 nm with a spectral resolution of 3.0 nm. A sensor measures the light signal supplied by a fiber optical probe, which collects light reflected by a 10x10 cm2 Spectralon white plate. The plate was held at about 1 m above the surface and was directed first towards the measured surface and then towards the sky. The ratio of these two measurements gives the hemispherical reflectance (albedo) of the surface. For some cases the water depth and ice thickness within the pond were measured.

For the model verification we considered the melt pond albedo in the spectral interval 0.25 – 1.3 μm. The retrieval procedure implies searching for the final parameters values shown in Table 1. These three parameters comprise a 3D-vector, which is varied to provide the best fit (in the sense of the least squares) of the measured and modeled spectra. For the cases where the pond depth and underlying ice thickness were measured, the pond parameters retrieved were compared to the measured ones.

Some ponds were frozen over, i.e., they had a layer of newly formed ice on top of their surface. It is evident that a layer of flat, transparent ice at the pond surface practically does not change pond reflection, so we consider the ponds with ice crust in the same manner as open ones. However, if the upper ice layer is hilly or snow-covered, the pond reflectance can change drastically; the pond gets brighter and may become indistinguishable from the surrounding ice in the visible range. These “bright” or snow-covered ponds would require other means for their characterization. We exclude such cases from consideration.

Figures 7-10 present photos of different ponds and their reflectance spectra, measured and retrieved with the presented algorithms (denoted as “retrieved” in the legend).

Figure 7 shows photos, modeled and measured spectra of light blue melt ponds which have a uniform bottom on their first year ice under clear and cloudy skies, measured in the Central Arctic on 10.08.2012, 19.08.2012 and 22.08.2012, respectively. The albedo values are extraordinary high. This could be related to the fact that the ponds are frozen over with a 2-3 cm layer of ice, which is likely not perfectly transparent. Figure 8 shows those cases of frozen over blue ponds with heterogeneous bottom over oceanic ice, measured on 31.08.12, 22.08.12 and 22.08.12, respectively. One can see darker parts in the ponds, which result from sea ice melting from the lower boundary or lower bubble content in regions of the pond bottom. Figure 9 presents dark open ponds on thicker first year ice under overcast skies, all measured on 26.08.2012. The albedo of these ponds is much lower than that of the previous ones: from about 0.07 to 0.14 in the visible and about 0.05 in the IR. Figure 10 presents two cases of light blue ponds, both measured on 20.08.12 and a dark pond contaminated with algae aggregates measured on 21.06.2012, all under overcast skies. Surprisingly, the spectrum of the pond with algae is reproduced quite well. This is because the contribution of the yellow algae spots to a total reflection is proportional to their
taken every four days. The most intensive formation process was observed from July 17 through August 14. The spectra taken during this period were processed and the results are shown in Figs. 12 and 13.

Figure 12 shows the spectra and the photos of the SHEBA dual pond. For the first five dates (July 17, 21, 23, 25, August 2) the retrieval is excellent (for the visible range RMSE = 0.0013 for July 17 and has a maximal value of 0.0086 for July 29, see Table 2) and for the last three (August 6, 11, 14) the retrieval is a little bit worse, but still quite good (for the visible range RMSE = 0.0085 for August 6 and 11). The reason for this difference is not obvious and we may assume that some contaminant got into the pond these days. So, the regression analysis relies on the first five measurement dates.

Figure 13 presents the retrieved pond depth and ice thickness (for both parts independently) for these dates. The retrieved pond depth at the light part is 7 cm greater and at the dark one in 12 cm greater than the average reported pond depth (37 cm). Albedo of the light part (in the visible part of spectrum) is approximately twice greater than that of dark part. In general, this agrees with the different nature of their physical properties. The retrieved ice thickness in the light part is lower by 6 cm in average than that of the dark part. The slope of the linear regression for the retrieved ice thicknese gives the melt rate of 1.9 cm/day and 2.6 cm/day for the light and dark parts, respectively. Taking the average surface and bottom melt for SHEBA ponded ice from 17 July to 14 August gives an estimated surface ice melt of 33 cm and bottom melt of 28 cm for a total of 63 cm, which gives a melt rate of 2.23 cm/day (Prosipich et al., 2003).

Table 2 shows the measured and the estimated pond depths and ice thicknesses. The retrieved melt depth at the light part is 7 cm greater in average than that of the dark part. The slope of the linear regression for the retrieved ice thickness gives the melt rate of 1.9 cm/day and 2.6 cm/day for light and dark parts, respectively. Taking the average surface and bottom melt for SHEBA ponded ice from 17 July to 14 August gives an estimated surface ice melt of 33 cm and bottom melt of 28 cm for a total of 63 cm, which gives a melt rate of 2.23 cm/day (Prosipich et al., 2003).

Suppose that the difference between the transport scattering coefficient $\sigma_t$ for the light and dark portion is due to air bubbles. Then, the scattering coefficient by air bubbles can be calculated as:

$$\sigma_t = \frac{\sigma_{bb}}{1 - \sigma_{bb}}$$

For the first five dates the average retrieved scattering coefficient by air bubbles is 33 m$^2$, the slope being much less than the scatter. In the bubble-scattered ice observed by Govorilo and Gritschenki (1973) the air volume concentration was up to 5% and the effective bubble radius was $R_b = 1.3$ mm. If we suppose the same effective radius, the average air volume concentration in the light ice will be $C_{bb} = 3/34 R_b = 2.8\%$, which is quite reasonable for bubbly ice.

### 4.4 Verification results

The retrieved and measured pond parameters (ice water depth and underlying ice thickness), as well as root mean square difference (RMSSD) in the measured and simulated albedo spectra, are given in Table 2. The RMSSD is shown both for the whole spectrum and for the visible range ($A < 0.7\mu m$). The scatter plot of the retrieved pond parameters is shown in Figure 14. The retrieval of the underlying ice thickness is made with reasonable accuracy; the maximal error is 55%, the relative RMSSD is 57% and $R^2 = 0.56$. The retrieval of the pond depth is more uncertain: its value can differ up to 2 times from the measured one and RMSSD = 65%. This is to be expected, because the pond water depth has much less effect on the pond albedo than the underlying ice thickness. Nevertheless, the correlation for the entire dataset of the measured and retrieved pond depth values is quite high ($R^2 = 0.62$) and 70% of the retrieved values are inside the 50%-error range. The area, which is not very large, however, their effect can be clearly seen in the spectrum: the measured values are less than the modeled ones in the blue range (0.3-0.5 pm) and greater in the yellow-green (0.5-0.6 pm).

The above ponds are quite different from one another. They range from dark to very light blue in color, open and frozen over, clear and contaminated with organic matter. In spite of this, the model is able to reproduce the measured spectra in the visible region with high accuracy in all studied cases. The normalized square differences (RMSSD) between the measured and simulated spectra has the average value of 0.04 for the whole considered spectrum from 0.35 to 1.3 pm and 0.017 for the visible range up to 0.73 pm.

The retrieved and measured geometrical parameters of the ponds, as well as the RMSSD between the measured and simulated spectra, are presented in Table 2 and shown in Figure 14.

### 4.2 Barrow-2008

Melt pond spectra were observed near Utqiagvik, Alaska USA (formerly Barrow) in 2008 as part of the SIZONET program observing pond formation (Palacki et al., 2012). Observations were collected at sites approximately 1 km offshore from Nikolenski in the Chukchi sea near 71.346N, 156.542W as level, landfast first year ice. For this work, a total of 27 measured melt pond spectra were used (16 photographs were taken). All melt ponds were quite dark and polluted with sediments and their spectra look quite similar. Three of them are presented in Figure 11. The albedo does not exceed the value of 0.3 in its maximum and show a discrepancy in the blue range, presumably due to the presence of mineral sediments. Because of this, the RMSSD between the measured and simulated spectra for the visible range (0.03) is greater than that for the whole spectrum (0.008). The ice thickness was not measured. The pond depths, measured and retrieved, as well as the RMSSD, are shown in Table 2 and Figure 14.

### 4.3 SHEBA-1999

SHEBA was a year-long drift experiment conducted in the Beaufort Sea from October 1997 to October 1998 (Perovich et al., 1999; Tutul et al., 2002). Extensive measurements of the characteristics of sea ice were made. This included observations of the spatial variability and temporal evolution of the spectral albedos of the ice cover (Perovich et al., 2002).

One pond in this expedition was especially interesting, because its bottom had a region that was much brighter than the surrounding bottom. This region had sharp borders with rectangular corners (see the photo in Figure 12). This likely was a broken piece of bubbly multiyear ice that was incorporated into the ice cover. The piece of ice had more air bubbles than the darker adjacent ice. This dual pond was observed during the entire period of its formation and development. The most intensive pond formation process was observed from July 17 through August 14. The spectra were taken every four days during this period. The spectra processing results are shown in Figures 12 and 13.

Figure 12 shows the spectra and the photos of the SHEBA dual pond. For the first five dates (July 17, 21, 23, 25, August 2) the retrieval is excellent (for the visible range RMSSD = 0.0013 for July 17 and has a maximal value of 0.0061 for July 29, see Table 2) and for the last three (August 6, 10, 14) the retrieval is a little bit worse, but still quite good (for the visible
observed scatter in the retrieval results might partly be explained by the specifics of the field measurements of the water depth and ice thickness of the melt pond: ice drillings or water-depth measurements are performed at one single point of the melt pond and do not necessarily represent the average ice thickness or water-depth values which can be highly variable. Summarising the verification, we can say that the spectra retrieval in the visible range is good for all the considered cases.

Some difference is observed in the blue, where some colored organic matter or minor sediments are present in the melt water, and in the IR, where the reflectance is too low and the signal is noisy.

5 Conclusion

This work presents the optical model of melt ponds on sea ice. Similar to Malmström and Podgorny (2006) we assume a pond to be a plane-parallel layer of pure water on an ice substrate. We paid much attention to the pond bottom albedo as it is the main factor that determines the pond reflectance. The albedo of the ice substrate is calculated within the modified two-stream approximation (Goga et al., 1991), which relates the layer albedo to the transport scattering coefficient of the medium and its thickness. The analysis of the spectral behavior of the characteristics of the sea ice constituents (air bubbles and black inclusion) has shown that the average cosine of the scattering phase function, and therefore the transport scattering coefficient of sea ice, is spectrally neutral. Hence, the pond can be characterized by only three independent parameters that determine its reflectance through the visible and near IR-spectral range: the pond depth, the ice substrate thickness, and the ice transport scattering coefficient. The developed model proposes the simple analytical formulas to calculate the main reflective characteristics of a melt pond: the bidirectional reflection factor and the black and white sky albedo. The derivation of the analytical formulas becomes possible due to the assumption of the Lambert reflection by the pond bottom.

Although this assumption does not meet the reality in general, the model verification with the field measurements approves its reasonability, at least concerning the spectral albedo. Its validity for the pond BRF requires further investigations. Additional attention is paid to the correct account for the illumination conditions during the field measurements. It is shown that multiple reflections of light between the atmosphere and surrounding background can be neglected, so the a priori knowledge of the background albedo is not necessary. However, the sky conditions (overcast or clear, presence of clouds or aerosol load) should be specified to interpret the pond albedo as white, black, or blue sky albedo. In the last case, it is highly desirable to know the spectrally resolved atmospheric optical thickness for the field measurements.

The model presented was successfully used in the algorithm for the sea ice albedo and melt pond fraction retrieval from the MERIS data (Goga et al., 2015; Bonanomi et al., 2015a; Bonanomi et al., 2015b). The model provides accurate description of the melt pond reflective properties. It is robust and is applicable to a variety of melt pond type observed in the field. The presented model can be useful in the problems of physics of sea ice and in monitoring the melt of the Arctic and Antarctic sea ice cover. Moreover, it makes it possible to improve the parameterization of the underlying surface in

range $\text{RMSD} \leq 0.0085$ for August 6 and 10. The reason for this difference is not obvious and we may assume that some contaminant got into the pond these days. So, the regression analysis relies on the first five measurement days. Figure 13 presents the retrieved pond (depth) and ice thickness (for both parts independently) for these days. The retrieved pond depth at the light part of the pond is 7 cm greater than the average reported pond depth (37 cm) and 13 cm greater at the dark part. Albedo of the light part (for the visible part of spectrum) is approximately twice greater than that of the dark part. In general, this agrees with the different nature of the pond’s physical properties. The retrieved ice thickness in the light part is lower by 34 cm in average than that of the dark part. The slope of the linear regression for the retrieved ice thickness gives the melt rate of 1.9 cm/day and 2.6 cm/day for the light and dark parts, respectively. Taking the average surface and bottom melt for SHEBA period of ice from 17 July to 14 August gives an estimated surface ice melt of 35 cm and bottom melt of 28 cm for a total of 63 cm, which gives a melt rate of 2.25 cm/day (Borovoi et al., 2003).

Suppose that the difference between the transport scattering coefficient $\sigma_t$ for the light and dark part is due to air bubbles only, then the scattering coefficient by air bubbles can be estimated as

$$\sigma_t = \frac{\sigma_{t,\text{air}} - \sigma_{t,\text{avg}}}{1 - g_{t,\text{avg}}}$$  \hspace{1cm} (51)

The retrieved values, averaged for the first five dates, are the following: the transport scattering coefficient for the light part $\sigma_{t,\text{avg}} = 5.6 \text{ m}^{-1}$, for the dark part $\sigma_{t,\text{avg}} = 1.0 \text{ m}^{-1}$. The slope of the regression line for these five dates is much less than the values scatter. Using the value of 0.86 for $g_{t,\text{avg}}$, we found that the average retrieved scattering coefficient by air bubbles $\sigma_{t,\text{avg}}$ is $55 \text{ m}^{-1}$. In the bubble saturated ice observed by Groves and Golokhov (1970) the air volume concentration was up to 5% and the effective bubble radius was $R_b = 1.3 \text{ mm}$. If we suppose the same effective radius, the average air volume concentration in the light ice will be $C_{\text{a,b}} = 0.3/ \pi R_b^2 = 0.25 \%$, which is quite reasonable for bubbly ice.

2.4 Verification results

The retrieved and measured pond parameters (melt water depth and underlying ice thickness), as well as root mean square difference (RMSD) between the measured and simulated albedo spectra, are given in Table 2. The RMSD is shown both for the whole spectrum and for the visible range ($\lambda < 0.5 \mu \text{m}$). A scatter plot of the retrieved pond parameters is shown in Fig. 14. The retrieval of the underlying ice thickness is made with reasonable accuracy; the maximal error is 55%, the relative RMSD is 33% and $R^2 = 0.56$. The retrieval of the pond depth is more uncertain; its value can differ up to 2 times from the measured one and RMSD = 65%. This is to be expected, because the pond water depth has much less effect on the pond albedo than the underlying ice thickness. Nevertheless, the correlation for the entire dataset of the measured and retrieved pond depth values is quite high ($R^2 = 0.62$) and 70% of the retrieved values are inside the 50%-error range. The observed scatter in the retrieval results might partly be explained by the specifics of the field measurements of the water depth and ice
Data availability

The field data from the R/V Polarstern cruise ARK-XVI/3 are available at the PANGAEA data repository (Jostowsky et al., 2016-2017).

Acknowledgements

The authors are grateful to the scientific party of the ARK-XVI/3 cruise for making the spectral albedo measurements possible. Special thanks are expressed to M. Neumann for organizing the logistics and to the Ice Sea Physics group on board for assisting with the measurements.

References


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the melt pond reflective properties not only pond albedo, but also pond bi-directional reflectance, which is of great importance for processing remote sensing data, which is commonly measured under a specific, narrow incidence angle. Moreover, the approach presented can be easily extended to describe the light transmittance through sea ice, which is also important for the radiative budget of the Arctic Ocean. The model presented is able to reproduce a variety of melt pond types observed in the field. It can be applied to the problems of physics of sea ice and for monitoring the melt of specifically the Arctic melt pond covered and also the Arctic and Antarctic sea ice cover. Also, it makes it possible to improve the characterization of ponds radiative parameters in various atmospheric remote sensing retrievals over the Arctic summer sea ice (clouds, aerosols, trace gases) and potentially re-evaluate the climatic feedbacks and radiative budget of the Arctic region at a new accuracy level.

6 Data availability


The field data from the SHEBA-1996 expedition are available at a supplement to this manuscript.

Acknowledgements

The work was supported by the Institutional Strategy of the University of Bremen, funded by the German Excellence Initiative, and by the TRR 172 “Arctic Amplification: Climate Relevant Atmospheric and Surface Processes and Feedback Mechanisms (ACROSS),” funded by the German Research Foundation (DFG).

The authors are grateful to the scientific party of the ARK XXVIII cruise for making the spectral albedo measurements possible. Special thanks are expressed to M. Nickel for organizing the logistics and to the Sea Ice Physics group on board for assisting with the measurements.

References


Table 1. Melt pond characteristics

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<th>Variable characteristics</th>
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Pozhidaev, P. R.; Factor Controlling the Surface Energy Budget over Snow and Ice, Academic Dissertation in meteorology, Finnish Meteorological Institute, Helsinki, 2008;

Pozhidaev, P. R., Hiebeler, J., and Rasmussen, H. V.: Observation and determination of albedo decrease caused by melting Arctic sea ice. Proc. of the NAS of the USA, 111 (43), 15279-15284, doi:10.1073/pnas.1411929111, 2014;


Riedel, A., Klos, L., and Brümmer, G.: Melt ponds on Arctic sea ice determined from MODIS satellite data using an artificial neural network. The Cryosphere, 6, 431-440, 2012;


Table 2. Measured and retrieved pond geometric parameters

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Figure 1. Schematic image of light rays in a melt pond. \( I_0 \) is the intensity of the incident light, \( \theta_i \) is the angle of incidence, \( I_r \) is the intensity of light, reflected from the air-water interface (AW), \( I_t \) is the intensity of light, reflected by the AW interface, \( \theta_r \) is the angle of refraction, \( I_u \) is the intensity of the up-welling diffuse light, \( \theta_i' \) is the angle of internal reflection, \( I_t' \) is the intensity of light after internal reflection by the AW interface, \( \theta_i' \) is the angle of internal reflection, \( I_r' \) is the intensity of light that comes out of the melt pond after refraction by the AW interface, \( \theta_i' \) is the observation angle equal to the angle of refraction.

Table 1. Melt pond characteristics

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Figure 2. Spectra of the relative refractive index (water-to-air): distilled water (open), brine with zero salinity at 0°C (dashed), and brine with equilibrium salinity at different temperatures (solid).

Table 2. Measured and retrieved pond geometric parameters derived from the spectral range 0.25 to 1.3 μm (total).

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Figure 3: Phase functions of the mixture of air bubbles and brine inclusions at -2°C with different fraction ratios $C_{air}/C_{brine}$. The ratio of transport scattering coefficients $Q_{air}/Q_{brine}$ and the average cosine $g$ are also shown. The effective sizes are $R_{air}$=42.35 μm, $R_{brine}$=100 μm.

Figure 1: Schematic image of light rays in a melt pond. $I_0$ is the intensity of the incident light, $θ_i$ is the angle of incidence; $I_1$ is the intensity of light, reflected from the ice-water interface (AW); $I_1^{AW}$ is the intensity of light, reflected by the AW interface; $θ_1$ is the angle of reflection; $I_2^{AW}$ is the intensity of the up-welling diffuse light; $θ_2$ is the angle of internal reflection; $I_2$ is the intensity of light after internal reflection by the AW interface; $θ_2$ is the angle of internal reflection; $I_3$ is the intensity of light that comes out of the melt pond after reflection by the AW interface; $θ'$ is the observation angle usual to the angle of reflection.
Figure 6: Blended sky albedo of a light mud pond (z = 17 cm, σ = 3.2 m/s, H = 1.25 m) vs. the angle of incidence (dashed). The white-
sky albedo values are shown in solid.

Figure 7: Spectra of the relative refractive index "water-ice", distilled water (symbols), fresh water at 0°C (dashed), and brine with
equilibrium salinity at different temperatures (solid).
Figure 5. Spectra of melt pond albedo at various illumination conditions and background albedo.

Figure 6. Phase functions of the mixture of air bubbles and brine inclusions with different fraction ratio $C_f/C_i$. The ratio of transmittance scattering coefficients $G'_f/G'_i$ and the average cosine $g$ are also shown. The effective sizes are $R_f=42.55 \mu m$, $R_i=100 \mu m$. 

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Figure 4. Blacksky albedo of a light ice pond (z = 1.7 cm, \( \sigma = 3.2 \text{m}^{-1} \), \( \theta = 1.25 \text{m} \)) vs. the angle of incidence (towards). The white-sky albedo values are shown in solid.

Figure 6. Spectral albedo of each pond: snow, and white ice. Light pond: depth is 26 cm; \( \sigma = 4 \text{m}^{-1} \); transport optical depth is 2; dark pond: depth is 20 cm; \( \sigma = 2 \text{m}^{-1} \); transport optical depth is 1; white ice: the effective grain size is 2 mm; optical depth is 12; snow: the effective grain size is 0.2 mm; optical depth is 280.
Figure 7: Light frozen blue ponds

Figure 8: Absorbance spectra of melt pond albedo at various sky conditions and background albedo at sun elevation 11°.
Figure 8: Frozen blue ponds

Figure 6: Typical spectral effects of melt ponds, snow, and white ice, calculated for the following parameters: light pond = depth $d$ is 20 cm, $\sigma_i = 4 \text{ m}^{-1}$, $H = 1.25 \text{ m}$, lower part optical depth is 3; dark pond = depth $d$ is 20 cm, $\sigma_i = 2 \text{ m}^{-1}$, $H = 0.5 \text{ m}$, lower part optical depth is 1; white ice = the effective grain size is 2 mm, optical depth is 12; snow = the effective grain size is 0.2 mm, optical depth is 200 (see Malinka et al., 2016, for details).
Figure 6: Dark open ponds

Figure 7: Light fronts (0-1 cm layer of ice) blue ponds.
Figure 10. From left to right: the light blue pond, a darker part of the blue pond, and the dark pond with yellow algae.

Figure 8. Frozen blue pond.
Figure 11. Spectra of some melt ponds from Baranov 2008: (left to right) a blue melt pond, a turbid blue melt pond, and a blue melt pond with brown algae.

Figure 9. Dark open ponds.
Figure 12: SREF/N dual pond: photos and spectra measured (dashed) at the light (blue) and dark (red) parts and simulated (solid). The photographs are taken at the early and late melt seasons (on June 3 and August 8, respectively).
Figure 12. Retrieved pond depth (left) and ice thickness (right) for the two parts of the dual pond shown in Fig. 12. The measured pond depth is shown with crosses. The dashed lines show the linear regression for the five 5 dates.

Figure II. Spectra of some melt ponds from Jarvis, 2009: (left to right) a blue melt pond, a bubbly blue melt pond, and a blue melt pond with brown spots.
Figure 14: Ice thickness and pond depth measured at different stations and retrieved. For ice thickness $R^2 = 0.85$ (N=8) and for pond depth $R^2 = 0.62$ (N=20).

Figure 15: SHEBA data: pond photos and spectra, measured (dashed) at the light (blue) and dark (red) parts and simulated (solid). The photographs are taken at the early and late melt season (July 3 and August 8, respectively).