Author’s response to reviews of: Simulating the roles of crevasse routing of surface water and basal friction on the surge evolution of Basin 3, Austfonna ice-cap
Yongmei Gong, Thomas Zwinger, Jan Åström, Bas Altena, Thomas Schellenberger, Rupert Gladstone, John C. Moore

February 25, 2018

Dear Editor and Reviewers

We have revised the manuscript ‘Simulating the roles of crevasse routing of surface water and basal friction on the surge evolution of Basin 3, Austfonna ice-cap’ in response to the reviews. This includes more description of the models, clarifying the methodology of crevasses map generating from the satellite image and modeled crevasses distribution validation, a few modifications in the discussion section as well as the suggested modification for the figures (attached).

A point by point response to each reviewer is given below, and were also posted as replies on the discussion. We respond to the specific referee points (in bold) below. Our replies are in normal black font, original manuscript quotes are in italic, and new text in the manuscript is in blue.

Thank you for your consideration.
Yongmei Gong, Thomas Zwinger, Jan Åström, Bas Altena, Thomas Schellenberger, Rupert Gladstone, John C. Moore

Response to anonymous referee 1

We thank Anonymous Referee 1 for his/her thorough review of our original manuscript. The referee requested some revisions and we have in general agreed that they were needed and carried them out.

General Comments:

Throughout the paper until near the end of the discussion, it was unclear to me whether the water routing to the bed through crevasses was a cause or consequence of the fast flow. The text seems to emphasize the role of water in facilitating fast flow, but the access of water to the bed via crevasses must be (at least partially) a consequence of the flow regime. I think I probably agree with the authors if they are arguing that the crevasses play an amplifying role, in that some reduction in basal traction is required to explain the formation of the crevasses that initially allow water to reach the bed. This water then accumulates in part of the domain and amplifies the acceleration of the outlet glacier. Though probably reported elsewhere, I found myself wanting to know if the thermodynamics work out: is there enough meltwater for this to be plausible given Austfonna’s thermal structure? The paper would be strengthened by a clear articulation of cause versus consequence.

Yes a good point. We realize that we need to clarify the role of the crevasses and consequentially, the surface melt water in the “hydro-thermodynamic” feedback. This corresponds both to the short
term seasonal speed up, and the role of the crevasses in the long term active surge phase. Certainly, the crevasses were initiated as a consequence of extensional flow that resulted from changes in basal thermal structure at the early stage of the active surge phase and were the triggering and enhancing factor of the ‘annual hydro-thermodynamic feedback’ cut in later on.

Verification of the “hydro-thermodynamic” feedback cannot be done in this study as we have neither the amount of the water reaching the bed nor a basal sliding relation engaging the basal effective pressure. However, the basal temperature distribution inversely calculated from the glacier geometry and velocity in Gong et al. (2016) has shown that the presence of a partially temperate bed in 1995 and the expansion of the temperate region from 1995 to 2011, which is consistent with the existence of basal melt water in the early stage of the surge active phase. Then calculated flow paths of both surface and basal melt water in 2012 correspond well with the fast flowing area, indicating the possible contribution of the basal water to the acceleration of the ice flow.

We modified the original text in Sec. 5:

from

“We agree that the so-called “hydro-thermodynamic” feedback proposed by Dunse et al. (2015) could explain the development of the surge in Basin 3 in general. Based on our results we now further present arguments to emphasize the role of crevasse formation, summer melt and basal hydrology system played in the seasonal speed-up events.”

to

“We cannot directly simulate or quantify the effects of the surface melt water or basal melt water on the surge development due to the lack of a basal effective pressure dependent sliding relation. However, based on our results we can still present arguments to emphasize the role of crevasse, summer melt and basal hydrology system in the seasonal speed-up events.”

and from

“In the end, our results support the “hydro-thermodynamic’ mechanism, in which crevasses provide access for surface melt water to reach the bed. We have demonstrated that cut-through crevasses are likely to be present approaching the surge in Basin-3, and that water flow paths route surface meltwater along flow paths corresponding to the regions of observed fast flow. While Dunse et al. (2015) are unspecific as to the cause of “hydro-thermodynamic” initiation zone, we propose that basal melt water, resulting from the build-up of the reservoir area and gradual thickening of ice (and hence raising of basal temperatures) during the quiescent phase, could sufficiently enhance flow speeds to initiate cut-through crevasses. Given that basal meltwater fluxes are likely to be at least an order of magnitude lower than surface meltwater or runoff fluxes, their impact on glacier sliding is likely to be much smaller. We suggest that basal meltwater, which is likely to be primarily routed toward the northern rather than southern flow unit due to topographic constraints (Fig. 6b), caused the speed up from the quiescent phase during the last part of the 20th century and early 21st century. This would require two key developments from quiescent to surge phase. Firstly, the initiation of sliding after ice thickening provided sufficient insulation for the bed to reach pressure melting temperature and generate sufficient meltwater, which could have occurred during the early nineties. Then at some point before August 2012 extensional flow due to sliding could have become sufficient to cause cut-through crevasses,
leading to further acceleration and the surge onset due to the annual “hydro-thermodynamic” feedback.

It is not clear at which point the “hydro-thermodynamic” feedback cut in, though it is likely to have first occurred in the northern flow unit, due to this unit’s earlier acceleration. We suggest that the “hydro-thermodynamic” feedback cut in for the southern unit in 2011 or early 2012 due to crevasses penetrating near the southern margin (Fig. 5a), rapidly causing the basin wide surge.”

Then we also discuss the role of the crevasses formation in the long term acceleration. These are initiated as a consequence of extensional flow resulting from changes in the basal thermal structure in an early post-quiescent phase and act as the triggering and enhancing factor in the so-called ‘annual hydro-thermodynamic feedback’ proposed by Dunse et al. (2015). While Dunse et al. (2015) are unspecific as to the cause of “hydro-thermodynamic” initiation zone in the long term glacier acceleration, we propose that basal melt water resulting from the gradual thickening of ice (raising basal temperatures) during the quiescent phase, could sufficiently enhance flow speeds to initiate cut-through crevassing. The basal temperature distribution inversely calculated from the glacier geometry and velocity (Gong et al., 2016) showed a partially temperate bed in 1995 and expansion of the temperate region from 1995 to 2011, which is consistent with the presence of water at the bed. Given that basal meltwater fluxes are likely to be at least an order of magnitude lower than surface meltwater or runoff fluxes, basal melt probably has a relatively small influence on glacier sliding. We suggest that water at the bed, which is likely to be primarily routed toward the northern rather than southern flow unit due to topographic constraints (Fig. 6b), caused the speed up from the quiescent phase during the last part of the 20th century and early 21st century.

This would require two key developments from quiescent to surge phase. Firstly, the initiation of sliding after ice thickening provided sufficient insulation for the bed to reach pressure melting temperature and generate meltwater. This could have occurred during the early nineties. Then at some point before August 2012, extensional flow due to sliding became sufficient to cause cut-through crevasses leading to further acceleration and the surge onset due to the annual “hydro-thermodynamic” feedback. We have demonstrated that cut-through crevasses are likely to be present just prior to the surge in Basin 3, and that surface meltwater can flow along the paths corresponding to the regions of observed fast flow.

It is not clear at which point the “hydro-thermodynamic” feedback cut in, though it is likely to have first occurred in the northern flow unit, due to this unit’s earlier acceleration. We suggest that the “hydro-thermodynamic” feedback cut in for the southern unit in 2011 or early 2012 due to crevasses penetrating near the southern margin (Fig. 5a), rapidly causing the basin wide surge.

Direct verification of the long term evolution of the surge active phase discussed above cannot be provided without quantification of the water reaching the bed and a basal sliding relation engaging the basal effective pressure. However our approach and results can throw some light on future studies of coupled ice dynamic/thermodynamic/hydrology simulations.”

Specific comments (page.line):

5.157-161: I read this several times and still have difficulty understanding how this procedure provides the validation data set.
We will need to validate the HiDEM simulation of crevasse locations by comparing them with the observational image of crevassing. This is a non-trivial exercise as the complete crevasse pattern is challenging to identify in optical imagery. Hence we used the following procedure to create an observational map of crevassing (Fig. 3c). We used both the orientation ($\theta$) of crevasse clusters and their response ($s(\theta)$) extracted from the Radon transformation with a line integral convolution. Later we use the Kappa statistical method to compare the similarity of the HiDEM and the observationally-based crevasse patterns.

We have modified the original text from:

‘To use the detected crevasse zones as a validation for our modeled crevasse distribution we transformed $s(\theta)$ and the orientation ($\theta$) into a cartographic representation (Fig. 3c). To do so, an empty image was randomly seeded with high intensities. Then a kernel with an elongated shape was convoluted over the image. This kernel was adaptive, as the orientation of the elongated shape is dependent on the orientation of the highest responding orientation signals in every window.’

to

“We wish to compare the simulated crevasse pattern from HiDEM with these results from the observation. To identify crevasse zones and their alignment in the satellite images we process an empty image array for each 300 m×300 m window with randomly seeded high intensity values. Then a simplified line integral convolution was applied to add each element of the image to its local neighbors, weighted by a kernel. The kernel has an elongated shape. The orientation of the shape is dependent on $\theta$ at the underlying position. The response of the kernel (the intensities within) was dependent on $s(\theta)$ extracted from the underlying position. The resulting image is shown in Fig. 3c, and will be compared with the modeled crevasses distribution visually as well as using the statistical Kappa method.”

5-6: For a technical journal like The Cryosphere, I was surprised not the see the model governing equations and instead a description of the model in prose. This is perhaps a matter of personal preference, but the methodology seems less ambiguous when described with the help of equations. Early on in the model description, it should be stated that sliding is implemented and that there is some kind of thermomechanical coupling (p 6, lines 191-192). It would be useful to know a bit more about the latter without having to read Gong et al (2016).

Agreed. We have modified/added the following text in Sec.3.1:

From L130: “The continuum ice dynamic model we used is Elmer/Ice, an open-source finite element model for computational glaciology. In this study, the simulations with Elmer/Ice were carried out by considering a gravity-driven flow of incompressible and non-linearly viscous ice flowing over a rigid bed. Some of the governing equations are presented below. More details can be found in Gagliardini et al.(2013).

The ice flow was computed by solving the unaltered full-Stokes equations, which express the conservation of linear momentum:

$$\nabla \cdot \sigma + \rho_i g = \nabla \cdot \tau - \nabla p + \rho_i g = 0, \quad (1)$$

and the mass conservation for an incompressible fluid:

$$\nabla \cdot u = \text{tr}(\dot{\varepsilon}) = 0, \quad (2)$$
in which \( \rho_i \) is the ice density, \( \mathbf{g} = (0, 0, -g) \) the gravity vector, \( \mathbf{u} = (u, v, w) \) the ice velocity vector, \( \sigma = \tau - p \mathbf{I} \) the Cauchy stress tensor with \( p = -\text{tr}(\sigma)/3 \) the isotropic pressure, \( \tau \) the deviatoric stress tensor, \( \mathbf{I} \) the identity matrix and \( \dot{\mathbf{e}} \) the strain-rate tensor.

The constitutive relation for ice rheology was given by Glen’s flow law (Glen, 1955):

\[
\tau = 2\mu \dot{\mathbf{e}},
\]

where the effective viscosity \( \mu \) is defined as

\[
\mu = \frac{1}{2} (EA)^{-\frac{1}{n}} \frac{1-n}{n} \dot{e}^2_e,
\]

in which \( n = 3 \) is the Glen’s flow exponent, \( \dot{e}^2_e = \text{tr}(\dot{\mathbf{e}}^2)\) is the square of the second invariant of the strain rate tensor; \( E \) is the enhancement factor; \( A \) is the rate factor calculated via Arrhenius law:

\[
A = A_0 \exp \left( -\frac{Q}{RT'} \right),
\]

\[
T' = T + \beta p,
\]

where \( A_0 \) is the pre-exponential constant, \( Q \) is the activation energy, \( R = 8.321 \text{ J mol}^{-1} \text{ K}^{-1} \) is the universal gas constant and \( T' \) is the temperature relative to pressure melting.

The upper surface, \( Z_s(x, y, z) \), evolves with time in transient simulations through an advection equation:

\[
\frac{\partial Z_s}{\partial t} + u_s \frac{\partial (Z_s)}{\partial x} + v_s \frac{\partial (Z_s)}{\partial y} - w_s = M_s,
\]

where \( (u_s, v_s, w_s) \) is the surface velocity vector obtained from the Stokes solution, \( M_s \) is the meteoric accumulation/ablation rate and \( s \) is the surface elevation.

For all the simulations carried out in this study a linear relation linking basal shear stress, \( \tau_b \), to basal velocity, \( \mathbf{u}_b = (u_b, v_b, w_b) \), is applied:

\[
\tau_b = -C \mathbf{u}_b,
\]

in which \( C = 10^6 \) is the basal friction coefficient.

We performed inverse modeling of basal friction coefficient distributions from all the surface velocity observation snapshots using Elmer/Ice based on the control method (MacAyeal, 1993; Morlighem et al., 2010), and implemented in Elmer/Ice by Gillet-Chaulet et al (2012). The inverse modeling determines the spatial distribution of the exponent, \( \alpha \), of the basal friction coefficient, \( C \), by minimizing the mismatch between modeled and observed surface velocity as defined by a cost function:

\[
J_0 = \int_{\Gamma_s} \frac{1}{2} \left( |\mathbf{u}_{mod}| - |\mathbf{u}_{obs}| \right)^2 d\Gamma,
\]

where \( |\mathbf{u}_{mod}| \) and \( |\mathbf{u}_{obs}| \) are the magnitude of the modeled and observed horizontal surface velocities. The mismatch in the direction of the velocity components is ignored. And only a match of velocity magnitude is optimized.

A Tikhonov regularization term penalizing the spatial first derivatives of \( \alpha \) is used to avoid over fitting:

\[
J_{reg} = \frac{1}{2} \int_{\Gamma_b} \left( \frac{\partial \alpha}{\partial x} \right)^2 + \left( \frac{\partial \alpha}{\partial y} \right)^2 d\Gamma,
\]

such that the total cost function is now written as:

\[
J_{tot} = J_0 + \lambda J_{reg},
\]
where $\lambda$ is a positive ad-hoc parameter. We adopted the same procedure as in Gillet-Chaulet et al. (2012) to find the optimal $\lambda$ value.

As introduced in Sect. 1, ideally, a soft-bed sliding mechanism needs to be presented in the simulation to be able to capture the surging behavior. However, as the main goal of this study is only to find a model approach to locate the surface melt water input sources, a linear basal sliding relation solved with an inverted parameter ($C$) which reflects the observation quite well (Fig. 2) is good enough to serve the purpose.

The temperature distribution is calculated according to the general balance equation of internal energy written as:

$$\rho_i c_v \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot (\kappa \nabla T) + D: \sigma,$$

(12)

where $\kappa = \kappa(T)$ and $c_v = c_v(T)$ are the heat conductivity and specific heat of ice, respectively. $D: \sigma$ represents the amount of energy produced by ice deformation. The upper value of the temperature $T$ is constrained by the pressure melting point $T_m$ of ice.

The Dirichlet boundary condition at the upper surface, $T_{surf}$, is prescribed as:

$$T_{surf} = T_{sea} + \Gamma z_s,$$

(13)

where $T_{surf}$ is the surface ice temperature, $T_{sea} = -7.68 \, ^\circ\text{C}$ is the mean annual air temperature at sea level estimated from two weather stations on Austfonna during 2004 and 2008 (Schuler et al., 2014) and four weather stations on Vestfonna during 2008 and 2009 (Möller et al., 2011), $\Gamma = 0.004 \, \text{K m}^{-1}$ is the lapse rate (Schuler et al., 2007).

An initial guess of the ice temperature, $T_{init}$, is given by:

$$T_{init} = T_{surf} + \frac{q_{geo}}{\kappa} d,$$

(14)

where $q_{geo} = 40.0 \, \text{mW m}^{-2}$ is the geothermal heat flux (Dunse et al., 2011) and $d$ the distance from the upper surface.

Spatially varied ice temperatures ($T$) snapshots in the flow solution were accommodated using an iterative process which includes four parts: i) Invert $C_{inert}$ for the first time with either an initial guess of $C_{init}$ and $T_{init}$ or the previously inverted $C_{prev}$ and $T_{prev}$; ii) Carry out steady state simulation for only thermodynamics to calculate $T_{inert}$ using the velocities obtained from the inversion; iii) Do the inversion again using $C_{inert}$ and $T_{inert}$ derived from the previous simulations; iv) Repeat the iteration until the differences in $C_{inert}$ and $T_{inert}$ between two successive iterations fall below a given threshold. More details about the interactive process can be found in Gong et al. (2016)."

And we have changed and moved the text in L169 – 176:

‘The criterion assumes that the calving front was always grounded with a positive height above floatation, which reflects the observation at the terminus in Basin 3. As the frontal and near-frontal region are not confined between lateral walls we would not expect significant impact of different calving front positions on longitudinal stress gradient upstream, i.e. the migration of calving front may have less impact on the basal shear stress distribution in the upstream area than the uncertainties brought by the observed ice velocity or the lack of ice thickness information at the calving front. On the other hand the basal shear stress calculation at the ice terminus will be effected. However the glacier bed is already very ‘slippery’ at the ice terminus. And as the ice front in the simulation did not advance the calving flux might be underestimated.’
“A fixed calving front criterion was adopted in all the simulations in this study due to the lack of ice thickness information corresponding to the observed calving front positions after 2011. The criterion assumed that the calving front was always grounded with a positive height above floatation, which reflected the observation at the terminus in Basin 3. As the near-frontal region was not confined between lateral walls we would not expect significant impact of different calving front positions on longitudinal stress gradient upstream, i.e. the migration of calving front would have less impact on the basal shear stress distribution in the upstream area than the uncertainties brought by the observed ice velocity or the lack of ice thickness information at the calving front. The fixed calving front criterion would not distort the basal shear stress calculation at the ice terminus neither, as the basal resistance there was already low in 2012. However as the ice front in the simulation did not migrate the calving flux might be biased.”

6: It becomes clear in the results and discussion that the HiDEM simulations do not include the change in stress state resulting from pre-existing crevasses, nor the advection of crevasses. Please emphasize these points in the methods: that the crevasses predicted by HiDEM reflect the stress field at a single snapshot in time, without consideration of any pre-existing damage or advection.

Yes, this is worth being more explicit about. Considering that the time step size in HiDEM is $10^{-4}$ s the modeled crevasses distribution does, somewhat, reflect the stress field instantaneously. Thus we added the following texts in Sec.3.2:

L225-232: “All the simulations in this study were carried out with 30 m spatial resolution (the particles are uniformly shaped and initially uniformly spaced). We used a time step length of $10^{-4}$ s, and ran a simulation until the glacier began to approach an equilibrium state. Compared to viscous flow, elastic deformation and fracturing processes are very rapid, and a typical simulation covers about ~ 10 minutes of glacier dynamics. At the end of a simulation, a crevasse field has formed. HiDEM reflects the instantaneous stress field calculated for the time of the input boundary conditions without consideration of any pre-existing damage or advection. Further details of the model, including sensitivity of the chosen parameters to the model results are discussed in Åström et al. (2013, 2014) and Riikilä et al. (2015). All parameters were set beforehand”

7.213-215: Suggest moving this to methods or deleting.

We agree that the original text in L213 – L215 is unnecessary. To make the reading more natural we modified the original L213 -223: ‘We investigate the evolution of basal friction using inverse modeling to determine $C$ from the observed surface velocity between April 2012 and July 2014, spanning the period of the Basin 3 peak surge velocities in January 2013. We focus on the lower region close to the terminus that is fully covered by TSX velocity observations.

To make the pattern of the $C$ distribution clearer we plotted the common logarithm of $C$ ($\log_{10} (C)$), instead of $C$ itself. Figure 4a shows a clear expansion of low friction area ($\log_{10} (C) \leq -3.5$) both inland and to the frontal region in the southern basin before the glacier enters the peak of the surge. In 2011 the low friction patches in the central and southern basin were disconnected from the inland region and also behind a stagnant terminus.

In April 2012, before the summer melt season, a low friction region also appeared in the southern corner, though still with a stagnant ice front. The low friction area of the northern flow unit slightly expanded to the south through the relatively flat frontal area. However, the fast flow did not expand
beyond the margin of the sub-glacial valley, which exited through the northern part of the calving front (Sect. 2.1; Fig. 4a), and might impose some restriction to the expansion of fast flow.’

L281 – 289: “Figure 4 shows the friction pattern of the region that is fully covered by TSX velocity observations between April 2012 and July 2014, spanning the period of the Basin 3 peak surge velocities in January 2013. To make the pattern of the C distribution clearer we plotted the common logarithm of C (\log_{10} (C)). Figure 4a shows a clear expansion of low friction area (\log_{10} (C) \leq -3.5) both inland and to the frontal region in the southern basin before the glacier enters the peak of the surge.

In 2011 the low friction patches in the central and southern basin were disconnected from the inland region and also lie behind a stagnant terminus. Before May 2012, the enlarged low friction area in both northern and southern glacier terminus did not expand across the flat glacier bed in between them, which might impose some topographic restriction to the expansion of the fast flow.”

7. section 4.2: It would be useful to know how changes in the fracture and/or bed penetration criteria for crevasse formation affect the mismatch between modelled and observed crevasse distributions. Were these parameters set to maximize agreement, or decided upon in advance without knowledge of the outcome?

Agree. We have added more model description in Section 3.2

“HiDEM is a model for fracture formation and dynamics. In HiDEM, an ice body is divided into discrete particles connected by massless beams. The version of HiDEM used here is purely elastic, rather than visco-elastic (Åström et al., 2013). The elastic version is sufficient for the purpose of locating fractures governed by glacier geometry and basal friction. If the initial state of a model glacier is out of elastic equilibrium, deformation within the ice will appear as a result of Newtonian dynamics.

The explicit scheme for simulating the Newtonian dynamics and the elastic modulus can be found in Riikilä et al. (2015). We use a Young’s modulus \( Y = 2.0 \text{ GPa} \) and a Poisson ratio \( \nu \approx 0.3 \) for the modeled ice here. The modeled ice fractures if the stress on a beam exceed a fracture stress criterion (stretching or bending). The fracture stress is \( \sim 1\text{MPa} \).

All the simulations in this study were carried out with 30 m spatial resolution (the particles are uniformly shaped and initially uniformly spaced). We used a time step length of \( 10^{-4} \text{ s} \), and ran a simulation until the glacier began to approach an equilibrium state. Compared to viscous flow, elastic deformation and fracturing processes are very rapid, and a typical simulation covers about \( \sim 10 \) minutes of glacier dynamics. At the end of a simulation, a crevasse field has formed. HiDEM reflects the instantaneous stress field calculated for the time of the input boundary conditions without consideration of any pre-existing damage or advection. Further details of the model, including sensitivity of the chosen parameters to the model results are discussed in Åström et al. (2013, 2014) and Riikilä et al. (2015). All parameters were set beforehand.

We then also modified the text in Section 4.2 to make the crevasse validation and cut through crevasses selection procedure clearer. First of all, we have double checked the modeled results from HiDEM that the width of the all the ‘fractures’ is larger than 0.055m. Therefore we did not actually eliminate any modeled fractures at the stage. We have changed the original L232-233:
‘We used a minimum fracture width of 0.05 m to identify a crevasse in HiDEM, which allowed us to keep most of the fractures across the whole model domain.’

to L298:

“All the fractures calculated by HiDEM are wider than 0.055m, of which we regard as crevasses in this study.”

Secondly, we compared all the modeled crevasses with the crevasses map generated from the satellite image not only the cut-through ones. We moved the texts originally in L237-239:

‘We defined cut-through crevasses as crevasses that penetrate through 2/3 ice depth and assume that they could cut through the full depth of ice if filled with water and potentially route surface melt water into the basal hydrology system vertically.’

to Section 4.3 Surface and basal water sources (L339 - 340):

“We defined cut-through crevasses as crevasses that penetrate through 2/3 ice-depth and assume that they could cut through the full depth of ice if filled with water and potentially route surface melt water into the basal hydrology system vertically.”

As suggested by the reviewer we also checked the Kappa coefficient when including the artificial crevasses. The 4.6 × 4.6 km smoothing window for re-sampling is used to maximize the agreement. We think visual comparison can judge the agreement. The statistical method is just used to give the reader the quantitative information. Thus we changed the original L240 – 250: ‘The crevasse distribution from C post was validated using the crevasse map generated from satellite observations acquired on 240 4th August 2013. The cartographic map of the crevasse detection (Fig. 3c) from the satellite observation was used for the validation. To estimate the statistical quality of the simulated crevasse field with the observationally estimated map we calculated the Kappa coefficient (K) (Wang et al., 2016). As almost any two maps will be significantly different with large sample size (> 62483) (Monserud and Leemans, 1992), we firstly re-sampled the two maps to an appropriate resolution. Experimentation leaded us to require a 4.6×4.6 km smoothing window to achieve substantial agreement (K = 0.71) (Cohen, 1960) between the maps. At higher resolutions K is worse for a variety of reasons: the ice dynamics model cannot advect crevasses, hence many crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasse densities are very variable and even at 1.5 km resolution the distribution is not smooth (K = 0.45); and the observationally derived map is not a perfect representation of reality. We next discuss the crevasse patterns derived from observations and those from the discrete element model in detail.’

to L317 – 326: “Although the visual comparison between the two maps shows a general agreement (Fig 5c), estimation of statistical quality of the simulated crevasse field with the observationally estimated map is necessary. We calculated the Kappa coefficient (K) (Wang et al., 2016) to quantify the agreement, but this is not trivial as almost any two maps will be significantly different with large sample size (> 62483) (Monserun and Leemans, 1992). We achieve moderate agreement (Cohen, 1960), (K = 0.45) when re-sampling the two maps with a 1.5 × 1.5 km smoothing window and substantial agreement (K = 0.71) with a 4.6 × 4.6 km smoothing window. When including the artificial crevasses (defined at the beginning of the section) the agreement is only fair (K ~ 0.30) for both re-sample windows. A variety of reasons can explain the resolution dependency of the results of the Kappa method: the ice dynamics model cannot advect crevasses, hence many
crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasse densities are very variable; and the observationally derived map is not a perfect representation of reality.”

7-8: It would really help to have some annotations of the figures to orient the reader to the geographical/morphological/dynamic regions of the domain that are referenced in the text (e.g. “margin of the subglacial valley”, “northern flow unit”). Perhaps a few numbers on the figures defined in the captions would do the trick.

Agreed. We marked SV (sub-glacial valley), OD (over-deepening area of the bed), NF (Northern flow unit) and SF (Southern flow unit) on Fig. 1b and added the following texts in the caption:

‘‘SV’’ marks the subglacial valley that runs between two bedrock maxima in the northeast and southwest and extends several tens of kilometers upstream and downstream. ‘OD’ marks the minimum bedrock height for Basin 3 and is within an over-deepening in the lower part of the valley. ‘NF’ marks the downstream area of the northern flow unit of the glacier, which runs from the upstream of the valley and exits from the northern terminus. The alignment of these labels roughly indicates the flow direction. Similarly, ‘SF’ marks the downstream area of the southern flow unit.’

In order to orient the readers to what we mean by saying ‘the crevasses above the margins of the sub-glacial valley’ we added two yellow boxes in Fig. 5a and the following text in the capitation:

“The red dots in the yellow boxes in (a) are referred to as cut-through crevasses above the sub-glacial valley margins and are used for calculating the flow paths of the surface melt reached the bed.”

8.244: Reword to state that the simulated and observed crevasse maps were resampled to maximize their correlation. An “appropriate resolution” would be one chosen based on the methodology and physical principles alone, rather than one chosen to maximize agreement.

Agreed. We think visual comparison is more sufficient to judge the agreement. The statistical method is just used to give the reader the quantitative information. Thus we changed the original L240 – 250 from: ‘The crevasse distribution from C_post was validated using the crevasse map generated from satellite observations acquired on 240 4th August 2013. The cartographic map of the crevasse detection (Fig. 3c) from the satellite observation was used for the validation. To estimate the statistical quality of the simulated crevasse field with the observationally estimated map we calculated the Kappa coefficient (K) (Wang et al., 2016). As almost any two maps will be significantly different with large sample size (> 62483) (Monserud and Leemans, 1992), we firstly re-sampled the two maps to an appropriate resolution. Experimentation leaded us to require a 4.6×4.6 km smoothing window to achieve substantial agreement (K = 0.71) (Cohen, 245 1960) between the maps. At higher resolutions K is worse for a variety of reasons: the ice dynamics model cannot advect crevasses, hence many crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasse densities are very variable and even at 1.5 km resolution the distribution is not smooth (K = 0.45); and the observationally derived map is not a perfect representation of reality. We next discuss the crevasse patterns derived from observations and those from the discrete element model in detail.’

to the texts below and put the them after the visual comparison:

L317 – 326 “Although the visual comparison between the two maps shows a general agreement (Fig 5c), estimation of statistical quality of the simulated crevasse field with the observationally
estimated map is necessary. We calculated the Kappa coefficient (K) (Wang et al., 2016) to quantify the agreement, but this is not trivial as almost any two maps will be significantly different with large sample size (> 62483) (Monserun and Leemans, 1992). We achieve moderate agreement (Cohen, 1960), (K = 0.45) when re-sampling the two maps with a 1.5 × 1.5 km smoothing window and substantial agreement (K = 0.71) with a 4.6 × 4.6 km smoothing window. When including the artificial crevasses (defined at the beginning of the section) the agreement is only fair (K = 0.30) for both re-sample windows. A variety of reasons can explain the resolution dependency of the results of the Kappa method: the ice dynamics model cannot advect crevasses, hence many crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasse densities are very variable; and the observationally derived map is not a perfect representation of reality.”

8.264-272: This paragraph seems more like discussion material.

Agreed. We modified and moved the original L264 – 272 in Sec. 4.2 Crevasses Distribution and Validation from: ‘This mismatch of the orientation between the modeled and observationally derived crevasse distribution in the middle upper area (Fig. 5c) may be due to HiDEM only simulating the ad-hoc formation and not advection of crevasses, thus no crevasse 265 formation history can be inferred from the model. The inclusion of crevasse advection could be implemented in a two-way coupling of HiDEM with a continuum model in future studies. The mismatch of the crevasse density (Fig. 5c) at the northern and southern frontal area could be caused by the mismatch of ice front position between the reality and the model. Although in reality the ice front advanced for several kilometers after the full-surge, it was kept fixed in position in Elmer/Ice (Sect. 3.1). The shape and steepness of the ice front likely affects the behavior of the discrete element model. However, as they are 270 concentrated at the terminus of the glacier, these crevasses are less likely to affect the basal hydrology system on a wider scale.’

to L372 – 379, Section 5 Discussion : ‘However there is a mismatch of the orientation in the middle upper area (Fig. 5c). It may be due to that HiDEM only simulates the ad-hoc formation but not the advection of crevasses, thus no crevasse formation history can be inferred from the model. The inclusion of crevasse advection could be implemented in a two-way coupling of HiDEM with a continuum model accounting for damage transport in future studies. The mismatch of the crevasse density (Fig. 5c) at the northern and southern frontal area could be caused by the mismatch of ice front position between reality and the model. Although in reality the ice front advanced for several kilometers after the full-surge, it was kept fixed in position in Elmer/Ice (Sect. 3.1). The shape and steepness of the ice front likely affects the behavior of the discrete element model. However, as they are concentrated at the terminus of the glacier, these crevasses are less likely to affect the basal hydrology system on a wider scale.’

Figure 3. Separate (a) and (b) a bit better, e.g. with a line or boxes. Figure 3 is scarcely mentioned in the text (bottom of pg 6) and no description appears to be given of the 4 panels on the right-hand side. Consider adding a sentence or two of explanation to the text.

Agreed. The former Fig.3 is now Fig.2. We added two rectangle frame in Fig. 2 to separate the results from August 2012 and August 2013. We also added more description of the figure in L276-280: “Figure 2 shows that the relative errors between the modeled and observed surface velocity magnitude for both the 18-29 August 2012 and the 16-27 August 2013 snapshots are the lowest over the fast flowing region (< 5%) (Fig. 2), the areas mostly moving by basal sliding. The root-mean-squared difference of the modeled surface velocity magnitude fields in the TXS data covered
region (Fig. 1) for these two time periods are 65.0 and 190.9 m a\(^{-1}\), respectively. As we are mostly interested in the ice dynamics of the fast flowing area, these errors are acceptable for the crevasse formation simulations.”

Technical corrections/queries (page.line):
The manuscript is clear and well-written overall, but still has some incorrect or awkward English phrasing. Articles (mostly “the”) are missing in multiple places throughout text, Thanks for pointing out. We have checked the language once again.

Response to anonymous referee 2
We thank Anonymous Referee 2 for their thorough review of our original manuscript. The referee suggested a number of modifications to the manuscript, and we have followed the advice given in the main.

General Comments:
The manuscript is generally well written, although it could benefit from some minor polishing for English grammar and sentence structure. The model descriptions are somewhat incomplete, such that it would not be possible to reproduce or confirm the results of this study. This can be easily addressed with additional text describing some of the explicit modelling choices.

We have checked the language once again and add more model descriptions in Section 3:

‘3.1 Basal friction inversion in the ice flow model
The continuum ice dynamic model we used is Elmer/Ice, an open-source finite element model for computational glaciology. In this study, the simulations with Elmer/Ice were carried out by considering a gravity-driven flow of incompressible and non-linearly viscous ice flowing over a rigid bed. Some of the governing equations are presented below. More details can be found in Gagliardini et al.(2013).

The ice flow was computed by solving the unaltered full-Stokes equations, which express the conservation of linear momentum:

\[ \nabla \cdot \sigma + \rho_i g = \nabla \cdot \tau - \nabla p + \rho_i g = 0, \]

(1)

and the mass conservation for an incompressible fluid:

\[ \nabla \cdot \mathbf{u} = \text{tr}(\dot{\varepsilon}) = 0, \]

(2)

in which \( \rho_i \) is the ice density, \( \mathbf{g} = (0,0,-g) \) the gravity vector, \( \mathbf{u} = (u,v,w) \) the ice velocity vector, \( \sigma = \mathbf{\tau} - p\mathbf{I} \) the Cauchy stress tensor with \( p = -\text{tr}(\sigma)/3 \) the isotropic pressure, \( \mathbf{\tau} \) the deviatoric stress tensor, \( \mathbf{I} \) the identity matrix and \( \dot{\varepsilon} \) the strain-rate tensor.

The constitutive relation for ice rheology was given by Glen’s flow law (Glen, 1955):

\[ \tau = 2\mu \dot{\varepsilon}, \]

(3)

where the effective viscosity \( \mu \) is defined as

\[ \mu = \frac{1}{2} (EA)^{-\frac{1}{n}} \left( \frac{1-n}{n} \right) \]

(4)
in which \( n = 3 \) is the Glen’s flow law exponent, \( \dot{\varepsilon}_e^2 = tr(\dot{\varepsilon}^2)/2 \) is the square of the second invariant of the strain rate tensor; \( E \) is the enhancement factor; \( A \) is the rate factor calculated via Arrhenius law:

\[
A = A_0 \exp\left(-\frac{Q}{RT}\right),
\]

\[
T' = T + \beta p,
\]

where \( A_0 \) is the pre-exponential constant, \( Q \) is the activation energy, \( R = 8.321 \text{ J mol}^{-1} \text{ K}^{-1} \) is the universal gas constant and \( T' \) is the temperature relative to pressure melting.

The upper surface, \( Z_s(x, y, z) \), evolves with time in transient simulations through an advection equation:

\[
\frac{\partial Z_s}{\partial t} + u_s \frac{\partial (Z_s)}{\partial x} + v_s \frac{\partial (Z_s)}{\partial y} - w_s = M_s,
\]

where \((u_s, v_s, w_s)\) is the surface velocity vector obtained from the Stokes solution, \( M_s \) is the meteoric accumulation/ablation rate and \( s \) is the surface elevation.

For all the simulations carried out in this study a linear relation linking basal shear stress, \( \tau_b \), to basal velocity, \( u_b = (u_b, v_b, w_b) \), is applied:

\[
\tau_b = -C \dot{u}_b,
\]

in which \( C = 10^\alpha \) is the basal friction coefficient.

We performed inverse modeling of basal friction coefficient distributions from all the surface velocity observation snapshots using Elmer/Ice based on the control method (MacAyeal, 1993; Morlighem et al., 2010), and implemented in Elmer/Ice by Gillet-Chaulet et al. (2012). The inverse modeling determines the spatial distribution of the exponent, \( \alpha \), of the basal friction coefficient, \( C \), by minimizing the mismatch between modeled and observed surface velocity as defined by a cost function:

\[
J_0 = \int_{\Gamma_s} \frac{1}{2} (|u_{mod}| - |u_{obs}|)^2 d\Gamma,
\]

where \(|u_{mod}|\) and \(|u_{obs}|\) are the magnitude of the modeled and observed horizontal surface velocities. The mismatch in the direction of the velocity components is ignored. And only a match of velocity magnitude is optimized.

A Tikhonov regularization term penalizing the spatial first derivatives of \( \alpha \) is used to avoid over fitting:

\[
J_{reg} = \frac{1}{2} \int_{\Gamma_b} \left( \frac{\partial \alpha}{\partial x} \right)^2 + \left( \frac{\partial \alpha}{\partial y} \right)^2 d\Gamma,
\]

such that the total cost function is now written as:

\[
J_{tot} = J_0 + \lambda J_{reg},
\]

where \( \lambda \) is a positive ad-hoc parameter. We adopted the same procedure as in Gillet-Chaulet et al. (2012) to find the optimal \( \lambda \) value.

As introduced in Sect.1, ideally, a soft-bed sliding mechanism needs to be presented in the simulation to be able to capture the surging behavior. However, as the main goal of this study is only to find a model approach to locate the surface melt water input sources, a linear basal sliding
relation solved with an inverted parameter (C) which reflects the observation quite well (Fig. 2) is good enough to serve the purpose.

The temperature distribution is calculated according to the general balance equation of internal energy written as:

$$ \rho_c c_v \left( \frac{\partial T}{\partial t} + u \cdot \nabla T \right) = \nabla \cdot (\kappa \nabla T) + D : \sigma, $$

where $\kappa = \kappa(T)$ and $c_v = c_v(T)$ are the heat conductivity and specific heat of ice, respectively. $D : \sigma$ represents the amount of energy produced by ice deformation. The upper value of the temperature $T$ is constrained by the pressure melting point $T_m$ of ice.

The Dirichlet boundary condition at the upper surface, $T_{surf}$, is prescribed as:

$$ T_{surf} = T_{sea} + \Gamma z_s, $$

where $T_{surf}$ is the surface ice temperature, $T_{sea} = -7.68 \, ^\circ C$ is the mean annual air temperature at sea level estimated from two weather stations on Austfonna during 2004 and 2008 (Schuler et al., 2014) and four weather stations on Vestfonna during 2008 and 2009 (Möller et al., 2011), $\Gamma = 0.004 \, \text{K m}^{-1}$ is the lapse rate (Schuler et al., 2007).

An initial guess of the ice temperature, $T_{init}$, is given by:

$$ T_{init} = T_{surf} + \frac{q_{geo}}{k} d, $$

where $q_{geo} = 40.0 \, \text{mW m}^{-2}$ is the geothermal heat flux (Dunse et al., 2011) and $d$ the distance from the upper surface.

Spatially varied ice temperatures ($T$) snapshots in the flow solution were accommodated using an iterative process which includes four parts: i) Invert $C_{invert}$ for the first time with either an initial guess of $C_{init}$ and $T_{init}$ or the previously inverted $C_{prev}$ and $T_{prev}$; ii) Carry out steady state simulation for only thermodynamics to calculate $T_{invert}$ using the velocities obtained from the inversion; iii) Do the inversion again using $C_{invert}$ and $T_{invert}$ derived from the previous simulations; iv) Repeat the iteration until the differences in $C_{invert}$ and $T_{invert}$ between two successive iterations fall below a given threshold. More details about the interactive process can be found in Gong et al. (2016).

All the thermodynamic-coupled inversions were done sequentially in chronological order with a transient simulation after each inversion to evolve the geometry for the next inversion. A month of geometry evolution starts with the C field inverted from the first velocity map acquired during that month to evolve the glacial geometry for 30 days with temporal resolution of half a day, and mean 1990-2000 surface mass budget (SMB) forcing from the regional climate model HIRHAM 5 (Christensen et al., 2007). In the case of acquisition time gaps (Table 1; mostly after August 2013) transient simulations were carried out for the length of the gap with the latest C distribution and temporal resolution of one day.

All simulations were computed on an unstructured mesh over Basin 3 generated with the open source software GMSH (Geuzaine and Remacle, 2009). The element size of the mesh increased from ~150 m at the glacier terminus to 2500 m at the back of the basin. The 2D mesh was then vertically extruded between the interpolated bedrock and surface elevation into 10 equally spaced terrain-following layers to form a three-dimensional (3D) mesh.
A fixed calving front criterion was adopted in all the simulations in this study due to the lack of ice thickness information corresponding to the observed calving front positions after 2011. The criterion assumed that the calving front was always grounded with a positive height above flotation, which reflected the observation at the terminus in Basin 3. As the near-frontal region was not confined between lateral walls we would not expect significant impact of different calving front positions on longitudinal stress gradient upstream, i.e. the migration of calving front would have less impact on the basal shear stress distribution in the upstream area than the uncertainties brought by the observed ice velocity or the lack of ice thickness information at the calving front. The fixed calving front criterion would not distort the basal shear stress calculation at the ice terminus neither, as the basal resistance there was already low in 2012. However as the ice front in the simulation did not migrate the calving flux might be biased.

3.2 Crevasse distribution calculation by a discrete element model

HiDEM is a model for fracture formation and dynamics. In HiDEM, an ice body is divided into discrete particles connected by massless beams. The version of HiDEM used here is purely elastic, rather than visco-elastic (Åström et al., 2013). The elastic version is sufficient for the purpose of locating fractures governed by glacier geometry and basal friction. If the initial state of a model glacier is out of elastic equilibrium, deformation within the ice will appear as a result of Newtonian dynamics.

The explicit scheme for simulating the Newtonian dynamics and the elastic modulus can be found in Riikilä et al. (2015). We use a Young’s modulus $Y = 2.0$ GPa and a Poisson ratio $\nu \approx 0.3$ for the modeled ice here. The modeled ice fractures if the stress on a beam exceed a fracture stress criterion (stretching or bending). The fracture stress is $\sim 1$ MPa.

All the simulations in this study were carried out with 30 m spatial resolution (the particles are uniformly shaped and initially uniformly spaced). We used a time step length of $10^{-4}$ s, and ran a simulation until the glacier began to approach an equilibrium state. Compared to viscous flow, elastic deformation and fracturing processes are very rapid, and a typical simulation covers about $\sim 10$ minutes of glacier dynamics. At the end of a simulation, a crevasse field has formed. HiDEM reflects the instantaneous stress field calculated for the time of the input boundary conditions without consideration of any pre-existing damage or advection. Further details of the model, including sensitivity of the chosen parameters to the model results are discussed in Åström et al. (2013, 2014) and Riikilä et al. (2015). All parameters were set beforehand.

The simulations were set up with input data from marine bathymetry, bedrock topography, $C$ field, and the surface topography. We selected two $C$ snapshots inverted from velocity data observed in 18-29 August 2012 ($C_{pre}$) and 16-27 August 2013 ($C_{post}$) (Fig. 2) as boundary condition for basal sliding in HiDEM. Those dates were chosen to model the crevasse distribution after the summer melt season before and after the peak in surge velocities observed in January 2013. The computations were carried out on an HPC cluster using typically 500 computing cores for a few hours.’

Specific comments:

L 27: “containing a marine-terminating…”

We have changed the original sentence ‘containing marine-terminating…’ to ‘containing a marine-terminating…’

L 70: awkward sentence, perhaps “previous crevasse modeling studies...”
We have changed the original sentence ‘Previous studies of modeling crevasse simulate…’ to ‘Previous crevasse modeling studies simulate…’

L 75, 198: discrete element models are not “first principle” models unless you are explicitly modeling a particulate medium. Glaciers are not composed of idealized spheres of ice connected in a lattice framework. These are model choices used to represent a certain class of phenomena, but such a model type does not arise inherently from first principles. This is not a criticism of the model itself, but it is misleading to consider it a first-principles model. Agree. We have deleted ‘first principle’ from both sentences.

L 87: “surge that occurred”

We have changed the original sentence from ‘…for the surge occurred in Basin 3’ to ‘…for the surge that occurred in Basin 3.’

L 92-94: You have here a ∼30 year discrepancy in time between the surface elevation model and the thickness observations used to create your bedrock map. Why not use the Moholdt and Kaab DEM that you later mention on L 110?

The bedrock map is not created using the surface elevation data acquired during July 2010 – December 2012. It was made by subtracting ice thickness (RES data from 1983 supplemented by two other data from 2008) from an older DEM (based on a Norwegian Polar Institute map published in 1998 and and InSAR data of Aust-fonna acquired in 1995–96. Its validity is discussed in Dunse., 2011. The bedrock elevation is used in other studies in addition to ours. In this study we assume bedrock elevation would not change in the time scale of decades then just simply updated the surface elevation to the data acquired during July 2010 – December 2012.

We realized that there is also a miswriting of the date when the older DEM was made. Thus we have changed the original text from

‘Surface elevation was derived from Cryosat altimetry data acquired during July 2010 – December 2012 (McMillan et al., 2014). McMillan et al., (2014) grouped measurements acquired over a succession of orbit cycles that are within 2-5 km2 geographic regions. Bedrock elevation (Dunse, 2011) was derived by point-wise subtracting the measured ice thickness from a 250 m resolution surface elevation that is based on the Norwegian Polar Institute (NPI) 1:250 000 topographic maps derived from aerial photography over Austfonna in 1983. The ice thickness used for generating bedrock elevation was based on airborne radio echo sounding (RES), (Dowdeswell et al., 1986) supplemented with two RES data sets from 2008 (Vasilenko et al., 2009). Marine bathymetry (2 km horizontal resolution) was from the International Bathymetry Chart of the Arctic Ocean, Version 2.0 (Jakobsson et al., 2008). Bathymetry and inland bedrock elevation were combined by using an interactive gridding scheme to eliminate the mismatch along the southern and northwest coast line (Dunse, 2011).’

to

“Bedrock elevation (Dunse, 2011) was derived by point-wise subtracting the measured ice thickness from a 250 m resolution surface elevation that is derived from a Norwegian Polar Institute (NPI) 1:250 000 topographic maps published in 1998 and InSAR data of Austfonna acquired in 1995-96 (Unwin and Wingham, 1997). The ice thickness used for generating bedrock elevation was based on airborne radio echo sounding (RES), (Dowdeswell et al., 1986) supplemented with two RES data sets from 2008 (Vasilenko et al., 2009). Marine bathymetry (2
km horizontal resolution) was from the International Bathymetry Chart of the Arctic Ocean, Version 2.0 (Jakobsson et al., 2008). Bathymetry and inland bedrock elevation were combined by using an interactive gridding scheme to eliminate the mismatch along the southern and northwest coast line (Dunse, 2011). We assume that bedrock elevation does not have any significant changes over decadal time scales, and use it with a set of updated surface elevation data. The surface elevation was derived from Cryosat altimetry data acquired during July 2010 – December 2012 (McMillan et al., 2014). McMillan et al. (2014) grouped measurements acquired over a succession of orbit cycles that are within 2-5 km² geographic regions.”

L 99-100: not a complete sentence

We have changed the original sentence from ‘The sub-glacial hill located at roughly 700 km E and 8850 km N rising to about 250 m a.s.l.’ to ‘The sub-glacial hill located at roughly 700 km E and 8850 km N rises to about 250 m above sea level.’

Section 2.3: this seems like it belongs more in the Methods section below, as it is not really “observations”

Agreed. We have moved the former ‘Section 2.3 Crevasse map’ to ‘Section 3.3 Crevasse map’ in ‘Section 3 Methodology’ and changed the order of the cited figures accordingly.

L 166: you mention modeling ice flowing over a rigid bed, but earlier mentioned that surge behaviour may result from ice flowing over a deformable bed. Perhaps worth commenting on this here?

As introduced in Sect.1, ideally, a soft-bed sliding mechanism needs to be simulated to be able to capture the surging behavior. However, as the main goal of this study is only to find a model approach to locate the surface melt water input sources, a linear basal sliding relation solved with an inverted parameter (C) which reflects the observation quite well (Fig. 2) is good enough to serve the purpose.

L 175: You mention the “slippery” terminus, but wasn’t the terminus the last to mobilize when the surge initiated? Maybe I’m missing something here, or perhaps you’re describing the terminus during the surge?

We meant to say that, in the present study, the fixed calving front criterion does not affect the basal shear stress at the ice front very much as the stagnant ice front has already disappeared at the time for which the simulations are carried out. We have changed the original sentence from ‘On the other hand the basal shear stress calculation at the ice terminus will be affected. However the glacier bed is already very ‘slippery’ at the ice terminus.’ to ‘The fixed calving front criterion would not distort the basal shear stress calculation at the ice terminus neither, as the basal resistance there is already low in 2012.’

Inversion routine: did you add any regularization in your inversions to prevent overfitting the observations? If so, how did you decide how much? If not, why? This seems to be an important point. Regularization is commonly (and appropriately) applied in this kind of work. If you used it, you need to describe it in detail here. If not, some justification of why not is needed.

Yes we did have regularization. The following sentences have been added in Section3.1

“A Tikhonov regularization term penalizing the spatial first derivatives of α is used to avoid overfitting:
\[ J_{reg} = \frac{1}{2} \int_{\Gamma_b} \left( \frac{\partial \sigma}{\partial x} \right)^2 + \left( \frac{\partial \sigma}{\partial y} \right)^2 d\Gamma, \]  

such that the total cost function is now written as:

\[ J_{tot} = J_0 + \lambda J_{reg}, \]  

where \( \lambda \) is a positive ad-hoc parameter. We adopted the same procedure as in Gillet-Chaulet et al. (2012) to find the optimal \( \lambda \) value.”

L 192-193: Ice temperatures are quite important in this kind of modeling. More description is needed here on how you computed spatially-varying temperature fields. Even if the details are in another reference, a general description of how you went about this is needed.

Agreed. We added the following sentences in Sect.3.1

‘The temperature distribution is calculated according to the general balance equation of internal energy written as:

\[ \rho c_v \left( \frac{\partial T}{\partial t} + u \cdot \nabla T \right) = \nabla \cdot (\kappa \nabla T) + D : \sigma, \]  

where \( \kappa = \kappa(T) \) and \( c_v = c_v(T) \) are the heat conductivity and specific heat of ice, respectively. \( D : \sigma \) represents the amount of energy produced by dice deformation. The upper value of the temperature \( T \) is constrained by the pressure melting point \( T_m \) of ice.

The Dirichlet boundary condition at the upper surface, \( T_{surf} \), is prescribed as:

\[ T_{surf} = T_{sea} + \Gamma z_s, \]  

where \( T_{surf} \) is the surface ice temperature, \( T_{sea} = -7.68 \, ^\circ C \) is the mean annual air temperature at sea level estimated from two weather stations on Austfonna during 2004 and 2008 (Schuler et al., 2014) and four weather stations on Vestfonna during 2008 and 2009 (Möller et al., 2011), \( \Gamma \) is 0.004 K m\(^{-1}\) is the lapse rate (Schuler et al., 2007).

An initial guess of the ice temperature, \( T_{init} \), is given by:

\[ T_{init} = T_{surf} + \frac{q_{geo}}{\kappa} d, \]  

where \( q_{geo} = 40.0 \, \text{mW m}^{-2} \) is the geothermal heat flux (Dunse et al., 2011) and \( d \) the distance from the upper surface.

Spatially varied ice temperatures (\( T \)) snapshots in the flow solution were accommodated using an iterative process which includes four parts: i) Invert \( C_{invert} \) for the first time with either an initial guess of \( C_{init} \) and \( T_{init} \) or the previously inverted \( C_{prev} \) and \( T_{prev} \); ii) Carry out steady state simulation for only thermodynamics to calculate \( T_{invert} \) using the velocities obtained from the inversion; iii) Do the inversion again using \( C_{invert} \) and \( T_{invert} \) derived from the previous simulations; iv) Repeat the iteration until the differences in \( C_{invert} \) and \( T_{invert} \) between two successive iterations fall below a given threshold. More details about the interactive process can be found in Gong et al. (2016).”

L 202: Elastic deformation is often considered static that is there is no time dependence. HiDEM is not a static model, but rather a dynamic model if you are using time stepping and accounting for dynamic stress propagation. Perhaps a semantic point...

'Elatic' is used as meaning reversible deformation - in opposite to e.g. 'Viscous' or 'Fractured'. Solutions in HiDEM naturally are computed transient.
L 203: typically a modeled glacier will approach a new equilibrium. . . (real glaciers do not produce crevasse fields in minutes!)

Crack propagation in ice is of the order of magnitude of a few 100m/sec - which means that a crevasse field can form in a few minutes.

L 204: See my comment above about reproducibility. A lot of choices are made when setting up a numerical model. For a study to be able to be reproduced/verified, you need to describe these choices. It’s okay to refer some general description in a reference, but there is essentially no detail on the HiDEM model implementation here. What kind of fracture criterion was used? How was the time stepping implemented? What kind of stopping criterion was used for the time stepping? How large are your discrete particles? Are they uniform in size and spacing? How sensitive are the model results to these choices? Would modifying any of these choices lead to better/worse agreement with the crevasse observations from this study?

Agreed. We have added more model descriptions in Section 3.2

“3.2 Crevasse distribution calculation by a discrete element model

HiDEM is a model for fracture formation and dynamics. In HiDEM, an ice body is divided into discrete particles connected by massless beams. The version of HiDEM used here is purely elastic, rather than visco-elastic (Åström et al., 2013). The elastic version is sufficient for the purposes of locating fractures governed by glacier geometry boundary and basal friction. If the initial state of a model glacier is out of elastic equilibrium, deformation within the ice will appear as a result of Newtonian dynamics.

The explicit scheme for simulating the Newtonian dynamics and the elastic modulus can be found in Riikilä et al. (2015). We use Young’s modulus $Y = 2.0$ GPa and the Poisson ratio $\nu \approx 0.3$ for the modeled ice here. The modeled ice fractures if the stress on a beam exceed a fracture stress criterion (stretching or bending). The fracture stress is $\sim 1$MPa.

All the simulations in this study were carried out with 30 m spatial resolution (the particles are uniformly shaped and initially uniformly spaced). We used a time step length of $10^{-4}$ sec, and ran a simulation until the glacier began to approach an equilibrium state. Compared to viscous deformation, elastic deformations are very rapid, and a typical typically simulation lasted $\sim 10$ minutes of simulated dynamics. At the end of a simulation, a crevasse field were formed. HiDEM reflect the instantaneous stress field calculated for the time of the input boundary conditions without consideration of any pre-existing damage or advection. Further details of the model, including sensitivity of the chosen parameters to the model results are discussed in Åström et al. (2013, 2014) and Riikilä et al. (2015). All parameters were set beforehand.”

Basal friction maps: some metric of the misfit (e.g. root mean square) would be useful to report to give an indication of the quality of the inversions. The misfit panels in Figure 3 have quite a lot of saturated regions on both the high and low ends of the misfit color scale. With these alone it is difficult to judge the quality of the fits.

Agreed. Most of the saturated regions on the high end are slow flowing regions. The relative errors in the fast flowing area are mostly below 5%. We also calculated the RMSD and added one paragraph at the beginning of the ‘Sec. 4.1 Basal friction evolution:
“Figure 2 shows that the relative errors between the modeled and observed surface velocity magnitude for both the 18-29 August 2012 and the 16-27 August 2013 snapshots are lowest over the fast flowing region (< 5%) (Fig. 2), the areas mostly moving by basal sliding. The root-mean-squared difference of the modeled surface velocity magnitude fields in the TXS data covered region (Fig. 1) for these two time periods are 65.0 and 190.9 m a⁻¹, respectively. As we are mostly interested in the ice dynamics of the fast flowing area, these errors are acceptable for the crevasse formation simulations.

It would be nice to see the evolution of velocities along with the friction evolution, for context.

Agreed. We have added observed speed snapshots in Figure 4.

L 232-233: I’m not sure what you mean by “keep” the fractures

We have double checked the modeled results from HiDEM that the width of the all the ‘fractures’ is larger than 0.055m. Therefore we did not actually eliminate any modeled fractures. We changed L232-233:

‘We used a minimum fracture width of 0.05 m to identify a crevasse in HiDEM, which allowed us to keep most of the fractures across the whole model domain.’

to L298:

“All the fractures calculated by HiDEM are wider than 0.055m, which we regard as crevasses in this study.”

L 235-235: the additional black region you mention is difficult to see in the figure. I’m not sure I see what you mean.

We realized that the description for the ‘black dots’ or rather all the dots in either the text or the caption of Fig.5 are quite ambiguous. Thus we have changed the original text from:

‘Many fractures were generated upstream of the sub-glacial hill (the area inside the black box in Fig. 5b); these were caused by boundary effects due to the limited domain and are excluded from the study (illustrated by 235 the region in Fig. 1a). A similar boundary effect causes incorrect crevassing in the southwest corner of the domain (also marked in black in Fig. 5b).’

to

“All the fractures calculated by HiDEM are wider than 0.055m, which we regard as crevasses in this study.”

L 235-235: the additional black region you mention is difficult to see in the figure. I’m not sure I see what you mean.

We realized that the description for the ‘black dots’ or rather all the dots in either the text or the caption of Fig.5 are quite ambiguous. Thus we have changed the original text from:

‘Many fractures were generated upstream of the sub-glacial hill (the area inside the black box in Fig. 5b); these were caused by boundary effects due to the limited domain and are excluded from the study (illustrated by 235 the region in Fig. 1a). A similar boundary effect causes incorrect crevassing in the southwest corner of the domain (also marked in black in Fig. 5b).’

to

“The fractures marked with black dots (Fig. 5b; in both upper left and lower left corner of the domain) are generated by boundary effects due to the limited domain. Although they might be deep enough to cut through the full depth of the ice we regard them as artificial crevasses. They are irrelevant to the water routing and surge processes we focus on in this paper thus are exclude from the comparison in this section and the water routing calculation in Sect. 4.3”

and also change the caption from:

‘Crevasses distribution from HiDEM on (a) August 2012 and (b) August 2013 and (c) satellite observation. The color of the underlying image in (a) and (b) shows the surface elevation of the glacier on land. Bedrock topography contours are shown in black with a ~23.7 m interval. White dots indicate the full modeled crevasse distribution in both (a) and (b). The superimposed red dots mark the cut-through crevasses. The Black box in (b) marks the area in which the fractures produced due to boundary effect (Sec. 4.2) are located. The superimposed black dots shown in (b) are eliminated from the crevasse map as they do not fulfil the definition of a crevasse in this study.'
The crevasse orientation of the satellite observation on 8 August 2013 is shown in (c) (color-coded with detecting intensity in the background). The magenta color shows the area where modeled and observed crevasse match. The basin side boundary is outlined with gray dashed line in all the sub-plots.’

‘Figure 5. Crevasse distribution from HiDEM on (a) August 2012 and (b) August 2013 and (c) satellite observation. The color of the underlying image in (a) and (b) shows the surface elevation of the glacier. Bedrock topography contours are shown in black with a ~23.7 m interval. All the dots in both (a) and (b), regardless of the color, indicate the modeled crevasse distribution from HiDEM. The red dots are the cut-through crevasses. The red dots in the yellow boxes in (a) are the ones referred as cut-through crevasses above the sub-glacial valley margins and are used for calculating the flow paths of the surface melt reached the bed. The black dots in (b) (upper left and lower left corner) mark crevasses produced due to boundary effects in the model (Sect. 4.2). They are eliminated from the crevasse map. The rest of the crevasses are marked with white dots, and are mostly shallow crevasses, hence irrelevant to water routing. The cartographic representation of the observed crevasse orientation on 8 August 2013 is shown in (c) (color-coded with detecting intensity in the background). The magenta color shows the area where modeled and observed crevasse match. The basin side boundary is outlined with gray dashed line in all the sub-plots.’

L 244: by "appropriate" you really mean you re-sampled until you got the best agreement.

This is not necessarily an objective "appropriate" resolution for comparing model output with observations (smoothing crevasses over 4.5 km kind of defeats the purpose of having discrete crevasses, doesn’t it?)

Agreed. We think visual comparison is more sufficient to judge the agreement. The statistical method is just used to give the reader the quantitative information. Thus we changed the original L240 – 250 from: ‘The crevasse distribution from C_post was validated using the crevasse map generated from satellite observations acquired on 240 4th August 2013. The cartographic map of the crevasse detection (Fig. 3c) from the satellite observation was used for the validation. To estimate the statistical quality of the simulated crevasse field with the observationally estimated map we calculated the Kappa coefficient (K) (Wang et al., 2016). As almost any two maps will be significantly different with large sample size (> 62483) (Monserud and Leemans, 1992), we firstly re-sampled the two maps to an appropriate resolution. Experimentation leaded us to require a 4.6×4.6 km smoothing window to achieve substantial agreement (K = 0.71) (Cohen, 245 1960) between the maps. At higher resolutions K is worse for a variety of reasons: the ice dynamics model cannot advect crevasses, hence many crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasse densities are very variable and even at 1.5 km resolution the distribution is not smooth (K = 0.45); and the observationally derived map is not a perfect representation of reality. We next discuss the crevasse patterns derived from observations and those from the discrete element model in detail.’

to the texts below and put the them after the visual comparison:

L317 – 326: “Although the visual comparison between the two maps shows a general agreement (Fig 5c), estimation of statistical quality of the simulated crevasse field with the observationally estimated map is necessary. We calculated the Kappa coefficient (K) (Wang et al., 2016) to quantify the agreement, but this is not trivial as almost any two maps will be significantly different
with large sample size (> 62483) (Monserun and Leemans, 1992). We achieve moderate agreement (Cohen, 1960), (K = 0.45) when re-sampling the two maps with a 1.5 × 1.5 km smoothing window and substantial agreement (K = 0.71) with a 4.6 × 4.6 km smoothing window. When including the artificial crevasses (defined at the beginning of the section) the agreement is only fair (K ~ 0.30) for both re-sample windows. A variety of reasons can explain the resolution dependency of the results of the Kappa method: the ice dynamics model cannot advect crevasses, hence many crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasse densities are very variable; and the observationally derived map is not a perfect representation of reality.”

L 261: 60 degrees is quite a mismatch, any comment on why this is the case here?

One explanation could be the modeled crevasses in HiDEM only reflects a material failure due the ‘instantaneous’ stress field which is dominated by extensional stress corresponding to the flow direction. But in reality there could be crevasses advected from upstream and got distorted on their way downstream, especially for the shallow crevasses in the middle upper area.

These are also discussed in discussion section, L373 – L375:

‘It may be due to that HiDEM only simulates the ad-hoc formation but not the advection of crevasses, thus no crevasse formation history can be inferred from the model. The inclusion of crevasse advection could be implemented in a two-way coupling of HiDEM with a continuum model accounting for damage transport in future studies.’

Panel 6c is mentioned in the caption of Figure 6, but not shown

We have changed the original caption from ‘Figure 6. The flow paths of different water sources derived from the results in August 2012. (a) The white lines indicate the path of surface melt water after entering the basal hydrology system via cut-through crevasses (red dots) according to hydraulic potential. The modeled basal velocity magnitude is color-coded in the background. (b) The white lines indicate the water path of basal melt water from locations with in-situ melt rates above 0.005 m a⁻¹. The bedrock elevation is color-coded in the background. (c) The logarithm (base 10) of the basal melt rate is color-coded in the background. The colored contour lines indicate the value of the basal melt rate. The black contour lines in both (a) and (b) indicate the hydraulic potential.’

to “Figure 6. The flow paths of different water sources derived from the results in August 2012. (a) The white lines indicate the path of surface melt water after entering the basal hydrology system via cut-through crevasses (red dots) according to hydraulic potential. The modeled basal velocity magnitude is color-coded in the background. (b) The white lines indicate the water path of basal melt water from locations with in-situ melt rates above 0.005 m a⁻¹. The colored contour lines indicate the value of the basal melt rate. The black contour lines in both (a) and (b) indicate the hydraulic potential.”

L 314: “factures” → “fractures”

The original sentence has been changed from ‘We used the discrete element model – HiDEM (Åström et al., 2014) to locate the possible location of crevasse fractures that...’ to ‘We used the discrete element model – HiDEM (Åström et al., 2014) to locate the possible location of crevasses.’

L 319: “emphasis” → “emphasize”
L 319-320: awkward sentence

The original paragraph has been changed from ‘We agree that the so-called “hydro-thermodynamic” feedback proposed by Dunse et al. (2015) could explain the development of the surge in Basin 3 in general. Based on our results we now further present arguments to emphasise the role of crevasse formation, summer melt and basal hydrology system played in the seasonal speed-up events.’

to “We cannot directly simulate or quantify the effects of the surface melt water or basal melt water on the surge development due to the lack of a basal effective pressure dependent sliding relation. However, based on our results we can still present arguments to emphasize the role of crevasse, summer melt and basal hydrology system in the seasonal speed-up events.”

Figure 4: some dates are cut off in the panels

We have fixed the problem.

References: check the reference list against those used in the text, it appears that a separate reference list has been concatenated to the end of the document (different font)

We have fixed the problem.
Simulating the roles of crevasse routing of surface water and basal friction on the surge evolution of Basin 3, Austfonna ice-cap

Yongmei Gong1, Thomas Zwinger2, Jan Åström2, Bas Altena3, Thomas Schellenberger3, Rupert Gladstone4, John C. Moore5

1Institute for Atmospheric and Earth System Research Division of Atmospheric Sciences, University of Helsinki, Helsinki, 00560, Finland
2CSC – IT Center for Science Ltd., Espoo, 02101, Finland
3Department of Geosciences, University of Oslo, Oslo, 0371, Norway
4Arctic Center, University of Lapland, Rovaniemi, 96100, Finland
5College of Global Change and Earth System Science, Beijing Normal University, Beijing, 100875, P.R. China

Correspondence to: Yongmei Gong (yongmei.gong@helsinki.fi)

Abstract. The marine-terminating outlet in Basin 3, Austfonna ice-cap has been accelerating since the mid-1990s. Step-wise multiannual acceleration associated with seasonal summer speed-up events was observed before the outlet entered its basin-wide surge in autumn 2012. We use multiple numerical models to explore hydrologic activation mechanisms for the surge behavior. A continuum ice dynamic model is used to invert basal friction coefficient distributions using the control method and observed surface velocity data between April 2012 and July 2014. This provides an input to a discrete element model capable of simulating individual crevasses, with the aim of finding locations where summer melt water enters the glacier and reaches the bed. The possible flow paths of input surface melt water at the glacier bed and basal melt water are calculated according to the gradient of the hydraulic potential. The inverted friction coefficients show the ‘unplugging’ of the stagnant ice front and expansion of low friction regions before the surge reached its peak velocity in January 2013. Crevasse distribution reflects to a high degree the basal friction pattern. The melt water reaches the bed through the crevasses located above the margins of the sub-glacial valley and the basal melt that is generated mainly by frictional heating flows either to the fast flowing units or potentially gets accumulated in an over-deepened region. Based on these results, the mechanisms facilitated by basal melt water production, surface melt water and crevasse opening, for the surge in Basin 3 are discussed.

1 Introduction

Austfonna ice-cap, located on Nordaustlandet in the Svalbard archipelago, is the largest ice mass in the Eurasian Arctic in terms of area (7800 km²) (Moholdt and Kääb, 2012). Basin 3 is one of its southeastern basins containing marine-terminating outlet glacier. The glacier is largely marine-grounded to as much as 150 m below sea level and is known to have surged around 1850-1870 (Dowdeswell et al., 1986).

The northern flow unit of the outlet glacier experienced long-term acceleration since the mid-1990s (Dowdeswell et al., 1986) along with stepwise inter-annual accelerations since 2008. These short-lived summer speed-up events occurred during the surface melt season (Dunse et al., 2015). The southern corner of Basin 3 accelerated to about 290 m a⁻¹ in spring 2008 but had decelerated again by spring 2011 (Gladstone et al., 2014). However high velocities were again observed in the same area...
during spring 2012 which subsequently gradually increased to ~1800 m a\(^{-1}\) after the summer melt season and before a basin-wide surge took place in autumn 2012 (Dunse et al., 2015). The surge reached its peak in January 2013 with a maximum velocity of ~6500 m a\(^{-1}\).

The 130-140 year quiescent phase of Basin 3 is similar to other Svalbard glaciers, but the two decades long accelerating phase of the northern flow unit exceeds those of other glaciers such as the 7-11 years of Monacobreen (Strozczi et al., 2002).

The step-wise multi-annual acceleration observed since 2008, associated with seasonal summer speed-up events, is also exceptional from other surging glaciers in Svalbard. Similar melt season speed-up events have been observed in Greenland, and provides evidence for rapid, large-scale, dynamic responses of the ice sheet to climate warming (Sundal et al., 2011; van de Wal et al., 2008; Zwally et al., 2002). Sundal et al (2011) pointed out that a simple model of basal lubrication alone could not simulate the fast flowing manner of the glaciers on Greenland ice sheet, and that an improved understanding of subglacial drainage would be essential for model studies to capture ice dynamic responses to climate warming. This applies also to the surge in Basin 3, which requires a mechanism involving both thermal and hydrologic changes to explain the inter-annual and seasonal accelerations (e.g. Dunse et al., 2015; Gladstone et al., 2014).

The glacier in Basin 3 (recently named Storisstrøumen) is polythermal, with a maximum ice thickness of 567 m, sufficient to raise internal ice temperatures to the pressure melting point (pmp) (Dunse et al., 2011). Where the ice is thinner, closer to the margins, the ice is probably frozen to the bed except under fast flowing outlets. In principle the surge of polythermal glaciers can be explained by a soft-bed mechanism with some constraints for the initiation, such as the unfreezing of the cold bed by the evolution of the thermal regime or by the input of melt water from englacial channels (Clark, 1976; Lingle and Fatland, 2003; Robin, 1955).

Gladstone et al. (2014) suggest soft-bed sliding mechanisms involving feedbacks in the hydrological system at the ice-till interface responding to penetration of surface melt to explain the summer speed-up events observed since 2008. Surface meltwater can penetrate cold and polythermal glacier ice in High Arctic glaciers and the Greenland ice sheet through moulins and fractures that cut down all the way to the glacier bed (e.g. Benn et al., 2009; Copland et al., 2003; van de Wal et al., 2008; Zwally et al., 2002). Water-filled crevasses can penetrate to the glacier bed regardless of ice thickness or crevasse spacing as long as the tensile stress acting normal to the crevasse exceeds about 100kPa (Boon and Sharp, 2003; van der Veen, 1998).

Bougamont et al (2014) investigated the sensitivity of the basal hydrology system in the Russell glacier catchment to the volume of surface melt delivered to the bed, finding increases in surface melt volumes lead to faster summer flow. Dunse et al. (2015) has suggested a “hydro-thermodynamic” feedback whereby summer meltwater penetrating to the bed is not considered a purely external forcing to the system: meltwater penetrating crevasses to reach the bed enhances basal processes such as lubrication and sediment deformation resulting in enhanced ice flow and potentially an increase in extensional stress, which may in turn cause increased crevasse formation over a wider area, routing more melt water down to the bed.

These earlier studies highlight the importance of time-evolving basal temperature and friction, which are strongly influenced by the evolution of a basal hydrology system. The basal hydrology system can be fed both by in situ melting and by surface meltwater, and has the capacity to not only directly cause sliding but also to alter the thermal regime and hence deformational flow.
Previous studies of modeling crevasse formation simulate the formation of fractures as a continuous process. They treat the development of cracks on a macroscopic scale by either using simplified parameterization of fracturing effects via variables such as depth of crevasse (Cook et al., 2014; Nick et al., 2010, 2013; Weertman, 1973) or using Continuous Damage Mechanics (CDM), which simulates the continuous process from micro-scale cracks to macro-scale crevasses (Albrecht and Levermann, 2014; Bassis and Ma, 2015; Borstad et al., 2012, 2016; Krug et al., 2014). In this study we take a different approach and apply a first-principles discrete element model (Åström et al., 2013, 2014) capable of simulating crevasse formation as a microscopic scale discrete process in addition to the continuum ice dynamics models. The discrete element model (HiDEM) is used to determine the locations of the crevasses penetrating though the full thickness of the glacier whereby surface water may reach the bed.

In Sect. 2 we present the observational data used for setting up the simulations and validating the results. In Sect. 3 we present the methodology. We use a continuum ice dynamic model to invert the basal friction field from approximately monthly observations of surface velocities between April 2012 and July 2014. This basal friction field then acts as a boundary condition for basal sliding in our discrete element model that simulates crevasse distribution in the lower part of Basin 3 for particular points in time. In Sect. 4.1 we firstly investigate the evolution of the basal conditions in Basin 3 during and after the peak of the surge. In sect. 4.2 we present the modeled crevasse distributions before and after maximum surge velocity and validate the latter with crevasse map derived from satellite imagery. In Sect. 4.3 we locate the crevasses that reach the bed, calculate basal melt rates and estimate the flow path of the basal water. In Sect. 5 we discuss the mechanisms facilitated by basal melt water production, surface melt water and crevasses opening for the surge that occurred in Basin 3. In Sect. 6 we summaries the key findings of the study and present conclusions.

2 Observations

2.1 Surface and bedrock topography data

Bedrock elevation (Dunse, 2011) was derived by point-wise subtracting the measured ice thickness from a 250 m surface elevation that is derived from a Norwegian Polar Institute (NPI) 1:250 000 topographic map published in 1998 and InSAR data of Austfonna acquired in 1995-96 (Unwin and Wingham, 1997). The ice thickness used for generating bedrock elevation was based on airborne radio echo sounding (RES; Dowdeswell et al., 1986) supplemented with two RES data sets from 2008 (Vasilenko et al., 2009). Marine bathymetry (2 km horizontal resolution) was from the International Bathymetry Chart of the Arctic Ocean, Version 2.0 (Jakobsson et al., 2008). Bathymetry and inland bedrock elevation were combined by using an interactive gridding scheme to eliminate the mismatch along the southern and northwest coast line (Dunse, 2011). We assume that bedrock elevation does not have any significant changes over decadal time scales, and use it with a set of updated surface elevation data. The surface elevation was derived from Cryosat altimetry data acquired during July 2010 — December 2012 (McMillan et al., 2014). McMillan et al. (2014) grouped measurements acquired over a succession of orbit cycles that are within 2-5 km² geographic regions.

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2.2 Surface velocity data

We used velocity time series maps (April 2012-July 2014) generated from TerraSAR-X (TSX) satellite synthetic aperture radar (SAR) scenes, (Table 1; Schellenberger et al., 2017, in review) as the input surface velocity data for basal friction coefficient inversion. These maps were based on original 2m resolution TSX scenes provided by the German Aerospace Center (DLR) covering only the lower part of Basin 3 (Fig. 1a). To generate the final velocity maps for the times between two successive TSX images, which were geocoded using a DEM of Austfonna (Moholdt and Kääb, 2012), we needed to use displacement maps. The displacement maps between two consecutive acquisitions were determined using cross-correlation of the intensity images (Strozzi et al., 2002). The coverage of the TSX velocity was smaller than the model domain used by our ice dynamic model (Fig. 1a). Therefore we stitched the TSX data on top of two background velocity fields with larger coverage depending on the acquiring time. The TSX data derived during 19 April 2012 – 28 December 2012 was stitched with velocity snapshot from ERS-2 (European Remote Sensing Satellite 2) SAR observation acquired in March to April 2011 (Gladstone et al., 2014; Schäfer et al., 2014); and the TSX data derived after 28 December 2012 was stitched with velocity snapshot from Landsat-8 imagery acquired in April 2013. We then applied a row-wise recalulation of the velocity value for the grid points on the model mesh that were upstream from the TSX velocity map coverage (Fig. 1a) to create a smoother transition from TSX velocity map to the background velocity map. The recalculation was carried out by weighting the background velocity data and TSX velocity data according to the distance between the column indices of the targeting grid point and the column indices of the first grid point that had value from TSX velocity map on the same row.

The velocity recalculated for the upstream area was simply to avoid numerical instability that might appear at the boundary between the TSX and background velocities. So as not to bias the crevasses distribution calculation, we confined the discrete element model domain to a smaller region close to the ice front, which was fully covered by TSX velocity map and far away from this transition zone (Fig. 1a).

3.0 Crevasse map

3.1.1 Basal friction inversion in the ice flow model

The continuum ice dynamic model we used is Elmer/Ice, an open-source finite element model for computational glaciology (Gagliardini et al., 2013). In this study, the simulations with Elmer/Ice were carried out by considering a gravity-driven flow of incompressible and non-linearly viscous ice flowing over a rigid bed.

Some of the governing equations are presented below. More details can be found in Gagliardini et al (2013).
The ice flow was computed by solving the unaltered full-Stokes equations, which express the conservation of linear momentum:
\[ \nabla \cdot \sigma + \rho_i g = \nabla \cdot \tau - \nabla p + \rho_i g = 0, \] (1)
and the mass conservation for an incompressible fluid:
\[ \nabla \cdot \mathbf{u} = \text{tr}(\mathbf{A}) = 0, \] (2)
in which \(\rho_i\) is the ice density, \(g = (0,0,-g)\) the gravity vector, \(\mathbf{u} = (u,v,w)\) the ice velocity vector, \(\sigma = \tau - pI\) the Cauchy stress tensor with \(p = -\text{tr}(\sigma)/3\) the isotropic pressure, \(\tau\) the deviatoric stress tensor, \(I\) the identity matrix and \(\dot{\mathbf{A}}\) the strain-rate tensor.

The constitutive relation for ice rheology was given by Glen’s flow law (Glen, 1955):
\[ \tau = 2\mu \dot{\mathbf{A}}, \] (3)
where the effective viscosity \(\mu\) is defined as
\[ \mu = \frac{1}{2} (E A)^{1-n} \dot{\varepsilon}^2, \] (4)
in which \(n = 3\) is the Glen’s flow law exponent, \(\dot{\varepsilon}^2 = \text{tr}(\dot{\mathbf{A}}^2)/2\) is the square of the second invariant of the strain rate tensor; \(E\) is the enhancement factor; \(A\) is the rate factor calculated via Arrhenius law:
\[ A = A_0 \exp \left( -\frac{Q}{RT} \right), \] (5)
\[ T' = T + \beta p, \] (6)
where \(A_0\) is the pre-exponential constant, \(Q\) is the activation energy, \(R = 8.321 \text{ J mol}^{-1} \text{ K}^{-1}\) is the universal gas constant and \(T'\) is the temperature relative to pressure melting.

The upper surface, \(Z_s(x, y, z)\), evolves with time in transient simulations through an advection equation:
\[ \frac{\partial Z_s}{\partial t} + u_s \frac{\partial Z_s}{\partial x} + v_s \frac{\partial Z_s}{\partial y} - w_s = M_s, \] (7)
where \((u_s,v_s,w_s)\) is the surface velocity vector obtained from the Stokes solution, \(M_s\) is the meteoric accumulation/ablation rate and \(s\) is the surface elevation.

For all the simulations carried out in this study a linear relation linking basal shear stress, \(\tau_b\), to basal velocity, \(\mathbf{u}_b = (u_b,v_b,w_b)\), was applied:
\[ \tau_b = -C \mathbf{u}_b, \] (8)
in which \(C = 10^6\) is the basal friction coefficient.

We performed inverse modeling of basal friction coefficient distributions from all the surface velocity observation snapshots using Elmer/Ice based on the control method (MacAyeal, 1993; Mortimer et al., 2010) implemented in Elmer/Ice by Gillet-Chaulet et al (2012). The inverse modeling determined the spatial distribution of the exponent, \(n\), of the basal friction coefficient, \(C\), by minimizing the mismatch between modeled and observed surface velocity as defined by a cost function:
\[ f_s = \int_{\Gamma_s} \frac{1}{2} (|u_{\text{mod}}| - |u_{\text{obs}}|)^2 \, d\Gamma, \] (9)
where \(|u_{\text{mod}}|\) and \(|u_{\text{obs}}|\) are the magnitude of the modeled and observed horizontal surface velocities. The mismatch in the direction of the velocity components is ignored. And only a match of velocity magnitude is optimized.

A Tikhonov regularization term penalizing the spatial first derivatives of \(n\) was used to avoid over fitting:
\( J_{\text{reg}} = \frac{1}{2} \int_{\Gamma} \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 d\Gamma \) \tag{10}

such that the total cost function is now written as:

\( J_{\text{tot}} = J_0 + \lambda J_{\text{reg}} \) \tag{11}

where \( \lambda \) is a positive ad-hoc parameter. We adopted the same procedure as in Gillet-Chaulet et al. (2012) to find the optimal \( \lambda \) value.

As introduced in Sect. 1, ideally, a soft-bed sliding mechanism needs to be presented in the simulation to be able to capture the surging behavior. However, as the main goal of this study is only to find a model approach to locate the surface melt water input sources, a linear basal sliding relation solved with an inverted parameter (\( C \)) which reflects the observation quite well (Fig. 2) is good enough to serve the purpose.

The temperature distribution was calculated according to the general balance equation of internal energy written as:

\[ \rho c_v \left( \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T \right) = \nabla \cdot \left( \kappa \nabla T \right) + D: \sigma \] \tag{12}

where \( \kappa = \kappa(T) \) and \( c_v = c_v(T) \) are the heat conductivity and specific heat of ice, respectively. \( D: \sigma \) represents the amount of energy produced by ice deformation. The upper value of the temperature \( T \) is constrained by the pressure melting point \( T_m \) of ice.

The Dirichlet boundary condition at the upper surface, \( T_{\text{surf}} \), is prescribed as:

\[ T_{\text{surf}} = T_{\text{sea}} + \Gamma z_s \] \tag{13}

where \( T_{\text{surf}} \) is the surface ice temperature, \( T_{\text{sea}} = -7.68 \, ^\circ C \) is the mean annual air temperature at sea level estimated from two weather stations on Austfonna during 2004 and 2008 (Schuler et al., 2014) and four weather stations on Vestfonna during 2008 and 2009 (Möller et al., 2011), \( \Gamma = 0.004 \, \text{K m}^{-1} \) is the lapse rate (Schuler et al., 2007).

An initial guess of the ice temperature, \( T_{\text{init}} \), was given by:

\[ T_{\text{init}} = T_{\text{surf}} + \frac{q_{\text{geo}}}{\kappa} d \] \tag{14}

where \( q_{\text{geo}} = 40.0 \, \text{mW m}^{-2} \) is the geothermal heat flux (Dunse et al., 2011) and \( d \) the distance from the upper surface.

Spatially varied ice temperatures (\( T \)) snapshots in the flow solution were accommodated using an iterative process which includes four parts: i) Invert \( C_{\text{init}} \) for the first time with either an initial guess of \( C_{\text{init}} \) and \( T_{\text{init}} \) or the previously inverted \( C_{\text{prev}} \) and \( T_{\text{prev}} \); ii) Carry out steady state simulation for only thermodynamics to calculate \( T_{\text{prev}} \) using the velocities obtained from the inversion; iii) Do the inversion again using \( C_{\text{prev}} \) and \( T_{\text{prev}} \) derived from the previous simulations; iv) Repeat the iteration until the differences in \( C_{\text{prev}} \) and \( T_{\text{prev}} \) between two successive iterations fall below a given threshold. More details about the interactive process can be found in Gong et al. (2016).

All the thermodynamic-coupled inversions were done sequentially in chronological order with a transient simulation after each inversion to evolve the geometry for the next inversion. A month of geometry evolution starts with the C field inverted from the first velocity map acquired during that month to evolve the glacial geometry for 30 days with temporal resolution of half a day, and mean 1990-2000 surface mass budget (SMB) forcing from the regional climate model HIRHAM 5 (Christensen et al., 2007). In the case of acquisition time gaps (Table 1; mostly after August 2013) transient simulations were carried out for the length of the gap with the latest C distribution and temporal resolution of one day.

All simulations were computed on an unstructured mesh over Basin 3 generated with the open source software GMSH (Geuzaine and Remacle, 2009). The element size of the mesh increased from \(-150 \, \text{m} \) at the glacier terminus to \( 2500 \, \text{m} \) at the
back of the basin. The 2D mesh was then vertically extruded between the interpolated bedrock and surface elevation into 10 equally spaced terrain-following layers to form a three-dimensional (3D) mesh.

A fixed calving front criterion was adopted in all the simulations in this study due to the lack of ice thickness information corresponding to the observed calving front positions after 2011. The criterion assumed that the calving front was always grounded with a positive height above floatation, which reflected the observation at the terminus in Basin 3. As the near-frontal region was not confined between lateral walls we would not expect significant impact of different calving front positions on longitudinal stress gradient upstream, i.e. the migration of calving front would have less impact on the basal shear stress distribution in the upstream area than the uncertainties brought by the observed ice velocity or the lack of ice thickness information at the calving front. The fixed calving front criterion would not distort the basal shear stress calculation at the ice terminus neither, as the basal resistance there was already low in 2012. However as the ice front in the simulation did not migrate the calving flux might be biased. The constitutive relation for ice rheology was given by Glen’s flow law (Glen, 1955), and the ice flow was computed by solving the Stokes equations. A fixed calving front criterion was adopted in all the simulations in this study due to the lack of ice thickness information corresponding to the observed calving front positions after 2011. The criterion assumes that the calving front was always grounded with a positive height above floatation, which reflects the observation at the terminus in Basin 3. As the frontal and near-frontal region are not confined between lateral walls we would not expect significant impact of different calving front positions on longitudinal stress gradient upstream, i.e. the migration of calving front may have less impact on the basal shear stress distribution in the upstream area than the uncertainties brought by the observed ice velocity or the lack of ice thickness information at the calving front. On the other hand the basal shear stress calculation at the ice terminus will be affected. However the glacier bed is already very ‘slippery’ at the ice terminus. And as the ice front in the simulation did not advance the calving flux might be underestimated.

3.3.2.3 Crevasse distribution calculation by a discrete element model

HiDEM is a model for fracture formation and dynamics. In HiDEM, an ice body is divided into discrete particles by massless beams. The version of HiDEM used here is purely elastic, rather than visco-elastic (Åström et al., 2013). The elastic version is sufficient for the purpose of locating fractures governed by glacier geometry and basal friction. If the initial state of a model glacier is out of elastic equilibrium, deformation within the ice will appear as a result of Newtonian dynamics.

The explicit scheme for simulating the Newtonian dynamics and the elastic modulus can be found in Riiikilä et al. use a Young’s modulus \( Y = 2.0 \) GPa and a Poisson ratio \( \nu = 0.3 \) for the modeled ice here. The modeled ice fractures if the stress on a beam exceed a fracture stress criterion (stretching or bending). The fracture stress is ~ 1MPa.

All the simulations in this study were carried out with 30 m spatial resolution (the particles are uniformly shaped and initially uniformly spaced). We used a time step length of \( 10^{-4} \) s, and ran a simulation until the glacier began to approach an equilibrium state. Compared to viscous flow, elastic deformation and fracturing processes are very rapid, and a typical simulation covers about ~ 10 minutes of glacier dynamics. At the end of a simulation, a crevasse field has formed. HiDEM reflects the instantaneous stress field calculated for the time of the input boundary conditions without consideration of any pre-existing damage or advection. Further details of the model, including sensitivity of the chosen parameters to the model results are discussed in Åström et al. (2013, 2014) and Riiikilä et al. (2015). All parameters were set beforehand.
The simulations were set up with input data from marine bathymetry, bedrock topography, C field, and the surface. We selected two C snapshots inverted from velocity data observed in 18-29 August 2012 (\(C_{pre}\)) and 16-27 August 2013 (\(C_{post}\)). Those dates were chosen to model the crevasse distribution after the summer melt season before and after the peak in surge velocities observed in January 2013. The computations were carried out on an HPC cluster using typically 500 computing cores for a few hours.

### 3.3 Crevasse map

We created a crevasse map from satellite imagery to validate our modeled crevasse distribution. The map was generated from a Landsat 8 image acquired on 4th August 2013 using the Radon-transform technique (Petrou and Kadyrov, 2004; Toft and Sørensen, 1996). We experimented with crevasse maps created from various different satellite sensors (Landsat 7, Landsat 8, ASTER, Sentinel-2), but here we used only the Landsat 8 scene which combines good spatial coverage with high radiometric quality. The Radon-transform has been demonstrated to be efficient in detecting along flow features (Roberts et al., 2013), but can also be used for complex flow patterns, like the one in Basin 3 which has a wide range of crevasse orientations. The advantages of the Radon-transform over other detecting methods are that crevasse patterns can be extracted where edge detectors methods (Bhardwaj et al., 2015; Wesche et al., 2013) would fail, and also that it is more robust than frequency-domain methods (Sangwine and Thornton, 1998) in detecting crevasses from incomplete coverage due to cloud coverage, image borders or the calving front.

In this study we followed a similar approach as Roberts et al. (2013), but used a more robust implementation and a different post-processing procedure. First, the satellite image was pre-processed with a Laplacian-filter to prioritize the high frequencies, e.g. to sharpen the edges of surface features. We performed the Radon-transform, \(R(p, \theta)\), on 300 m×300 m subsets of the satellite image, and project the image intensity \(I(x, y)\) along lines with tangent vectors oriented at \(\theta\) to the x-axis and offset by a perpendicular distance, \(p\), from the origin (Toft and Sørensen, 1996):

\[
R(p, \theta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y) \delta(x \sin \theta + y \cos \theta - p) \, dx \, dy
\]

where the 2D integration is restricted by the Dirac delta function, \(\delta(x \sin \theta + y \cos \theta - p)\), to the appropriate straight line in the x-y plane. The range of the transform coordinates is a half circle (\(0 \leq \theta < \pi\)) and \(p\) is the spatial integral ranging over the domain of the subset of the image. The result of the transform was a 2D feature space at different azimuthal orientations (\(\theta\)). To capture both small and big crevasses, we re-sampled the image intensity \(I(x, y)\) in each 300mx300m image subset with a resolution of 2 pixels and again implement a weighted Radon-transform function, where a mask over the subset was used to remove features like image borders, clouds, ocean etc. The resulting Radon transformation of a subset was again a two-dimensional subset. Then the standard deviation at each orientation was used to extract the response for elongated texture:

\[
s(\theta) = \frac{1}{N} \sum_{i=1}^{N} \frac{|R(i \theta - \bar{R}(\theta))|^2}{\bar{N} + 1}
\]

Here the overbar denotes the mean and \(N\) denotes the amount of steps within the domain of \(p\). Finally, a running median filter with a spacing of two \((\Delta = 2)\) was used to remove noise:

\[
\tilde{s}(\theta) = \text{median}(s(\theta - \Delta), \ldots, s(\theta + \Delta))
\]
The results of the procedure were maps showing the dominating azimuthal orientations (θ) of the crevasse clusters and their response (\(\tilde{s}(\theta)\)) (Fig. 3b) in each 300 m x 300 m window. We wish to compare the simulated crevasse pattern from HiDEM with these results from the observation. To identify crevasse zones and their alignment in the satellite images we process an empty image array for each 300 m x 300 m window with randomly seeded high intensity values. Then a simplified line integral convolution was applied to add each element of the image to its local neighbors, weighted by a kernel. The kernel has an elongated shape. The orientation of the shape is dependent on θ at the underlying position. The response of the kernel (the intensities within) was dependent on \(\tilde{s}(\theta)\) extracted from the underlying position. The resulting image is shown in Fig. 3c, and will be compared with the modeled crevasses distribution visually as well as using the statistical Kappa method.

HiDEM is a first-principle model for fracture formation and dynamics. The version of HiDEM we used here is a purely elastic model, rather than incorporating visco-elastic processes (Åström et al., 2013). This version is sufficient for the purposes of locating fractures given only geometric boundary conditions and basal friction coefficient. If the initial condition of HiDEM is out of equilibrium, elastic deformation within the ice will appear as a result of Newtonian dynamics. If local stress exceed a fracture criterion, the ice will begin to rupture. Compared to viscous deformation, elastic deformation is very rapid, and typically a glacier will approach a new equilibrium after some minutes of simulated dynamics. Consequently, at the end of a simulation, a crevasse field has been formed. A detailed description of HiDEM can be found in Åström et al. (2013, 2014).

### 16.4.1 Basal friction evolution

Figure 2 shows that the relative errors between the modeled and observed surface velocity magnitude for both the August 2012 and the 16-27 August 2013 snapshots are the lowest over the fast flowing region (< 5%; Fig. 2), the areas mostly moving by basal sliding. The root-mean-squared difference of the modeled surface velocity magnitude fields in the TXS data covered region (Fig. 1) for these two time periods are 65.0 and 190.9 m a\(^{-1}\), respectively. As we are mostly interested in the ice dynamics of the fast flowing area, these errors are acceptable for the crevasse formation simulations.

Figure 4 shows the friction pattern of the region that is fully covered by TSX velocity observations between April and July 2014, spanning the period of the Basin 3 peak surge velocities in January 2013. We investigate the evolution of basal friction using inverse modeling to determine \(C\) from the observed surface velocity between April 2012 and July 2014, spanning the period of the Basin 3 peak surge velocities in January 2013. We focus on the lower region close to the terminus that is fully covered by TSX velocity observations.

In 2011 the low friction patches in the central and southern basin were disconnected from the inland region and also lie behind a stagnant terminus. Before May 2012, the enlarged low friction area in both northern and southern glacier terminus did not expand across the flat glacier bed in between them, which might impose some topographic restriction to the expansion of the fast flow. In 2011 the low friction patches in the central and southern basin were disconnected from the inland region and also behind a stagnant terminus.

After January 2013 the basal friction pattern in northern basin remained almost stable. The almost vanishing friction area (log10 \(C\) ≤ -5.5) in the southern frontal region gradually shrank back inland away from the terminus.

### 17.4 Crevasse distribution and validation

All the fractures calculated by HiDEM are wider than 0.055m, of which we regard as crevasses in this study. The fractures marked with black dots (Fig. 5b; in both upper left and lower left corner of the domain) are generated by boundary effects.
due to the limited domain. Although they might be deep enough to cut through the full depth of the ice we regard them as artificial crevasses. They are irrelevant to the water routing and surge processes we focus on in this paper thus are exclude from the comparison in this section and the water routing calculation in Sect. 4.3. We used a minimum fracture width of 0.05 m to identify a crevasse in HiDEM, which allowed us to keep most of the fractures across the whole model domain. Many fractures were generated upstream of the sub-glacial hill (the area inside the black box in Fig. 5b); these were caused by boundary effects due to the limited domain and are excluded from the study (illustrated by the region in Fig. 1a). A similar boundary effect causes incorrect crevassing in the southwest corner of the domain (also marked in black in Fig. 5b). All of these artificial crevasses are irrelevant to the water routing and surge processes we focus on in this paper. We defined cut-through crevasses as crevasses that penetrate through 2/3 ice depth and assume that they could cut through the full depth of ice if filled with water and potentially route surface melt water into the basal hydrology system vertically.

The crevasse distribution from $C_A$ was validated using the crevasse map generated from satellite observations acquired on 4th August 2013. The cartographic map of the crevasse detection (Fig. 2c) from the satellite observation was used for the validation. To estimate the statistical quality of the simulated crevasse field with the observationally estimated map we calculated the Kappa coefficient ($K$) (Wang et al., 2016). At almost any two maps will be significantly different with large sample size (> 62483) (Monserud and Leemans, 1992), we firstly re-sampled the two maps to an appropriate resolution. Experimentation led us to require a 4.6×4.6 km smoothing window to achieve substantial agreement ($K = 0.71$) (Cohen, 1960) between the maps. At higher resolutions, $K$ is worse for a variety of reasons: the ice dynamics model cannot advect crevasses, hence many crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasses densities are very variable and even at 1.5 km resolution the distribution is not smooth ($K = 0.45$); and the observationally derived map is not a perfect representation of reality. We next discuss the crevasse patterns derived from observations and those from the discrete element model in detail.

The modeled crevasse distribution reflects the broad features of the basal friction pattern (Fig. 5b). A high crevasse density is generated in areas with large tensile stress caused by extending flow on the lower part of basin 3, as well as at shear margins between low and high friction areas. The orientation of the modeled crevasses above the sub-glacial valley margins agrees with the observation (Fig. 5b). However orientations of most of the modeled crevasses in the middle upper area have a ~60° mismatch with the satellite image (Fig. 5c) and the modeled crevasse density at the frontal area of the southern and northern flow units are larger than those in the observationally derived map.

Although the visual comparison between the two maps shows a general agreement (Fig. 5c), estimation of statistical quality of the simulated crevasse field with the observationally estimated map is necessary. We calculated the Kappa coefficient ($K$) (Wang et al., 2016) to quantify the agreement, but this is not trivial as almost any two maps will be significantly different with large sample size (> 62483) (Monserud and Leemans, 1992). We achieve moderate agreement (Cohen, 1960), ($K = 0.45$) when re-sampling the two maps with a 1.5 x 1.5 km smoothing window and substantial agreement ($K = 0.71$) with a 4.6 x 4.6 km smoothing window. When including the artificial crevasses (defined at the beginning of the section) the agreement is only fair ($K = 0.30$) for both re-sample windows. A variety of reasons can explain the resolution dependency of the results of the Kappa method: the ice dynamics model cannot advect crevasses, hence many crevasses in the image that in reality were created further upstream were simply not present in the simulation; crevasse densities are very variable; and the observationally derived map is not a perfect representation of reality. This mismatch of the orientation between the modeled
and observationally derived crevasse distribution in the middle upper area (Fig. 5c) may be due to HiDEM only simulating
To investigate the crevasse distribution after the summer melt season in 2012, we used C_{pre} and the corresponding geometry with HiDEM. The configuration produced more crevasses in the frontal region of the northern flow unit than in the southern flow unit and almost no crevasses over the frontal region of the central flow unit (white dots in Fig. 5a). Crevasses also appeared at the margins of the sub-glacial valley.

By looking at the overall crevasse distributions in August 2012 and August 2013 (white dots in Fig. 5a and 5b) together with their corresponding C distributions (Fig. 4) we noticed that the outline of the densely crevassed region more or less follows the outline of the low friction region, indicating the governing role of basal friction on crevasse formation. This was also shown by the fact that there were more crevasses formed in the southern and middle frontal area in August 2013 than in August 2012 as the bed was more ‘slippery’ in August 2013 (Fig. 4b). The confining effects of the bed rock topography to the fast flow, basal friction and crevasse distribution also became more visible in the later stage of the surge: the modeled crevasses at the sub-glacial valley’s sides indicated a sharper boundary in August 2013 than in August 2012.

17.2.4.3 Surface and basal water sources

We defined cut-through crevasses as crevasses that penetrate through 2/3 ice depth and assume that they could cut through the full depth of ice if filled with water and potentially route surface melt water into the basal hydrology system vertically.

We selected the cut-through crevasses in August 2012 and August 2013 (red dots in Fig. 5a and b) to identify possible routes of surface water to the bedrock. In August 2012 most of the crevasses in the frontal area cut through the ice deep enough and very likely represent future calving locations for the terminus during its advance. Most of the crevasses located between the northern and southern fast flowing regions were shallow, surface crevasses. Many crevasses above the margins of the sub-glacial valley could reach the bed and potentially route surface melt water from upstream to the bed. By August 2013 more cut-through crevasses had been developed in the lower southern and central basin compared with August 2012 as velocity gradients significantly increased after the basin-wide acceleration. There were more cut-through crevasses present above the shear margin but almost no cut-through crevasses above the over-deepening area.

Using the locations of cut-through crevasses above the margins of the sub-glacial valley that could potentially route surface melt water down to the bed, in August 2012 we calculated the sub-glacial water flow path according to the gradient of the hydraulic potential (Fig. 6a). The hydraulic potential (h) was calculated as below:

\[
\Delta h = (z_2 - z_3) \frac{\rho_i}{\rho_w} + z_b
\]

in which \(z_1\) and \(z_2\) are surface and bedrock elevation; \(\rho_i = 910\) kg m\(^{-3}\) and \(\rho_w = 1000\) kg m\(^{-3}\) are the density of ice and water.

The flow paths are generated by integrating through the vector field that follows the steepest descent in \(h\) using fourth-order Runge-Kutta method.

The surface melt water entering the bed at the north was predicted to either flow directly to the terminus, or stop at the sub-glacial over-deepening area (Sect. 2.1; Fig. 6a). Surface melt water entering the bed from the south was routed directly towards the terminus at the southern corner of the glacier, suggesting that surface melt contributed to the dramatic acceleration of the southern part of Basin 3 after the summer melt season in 2012.

In addition to the basal water supplied via the crevasse system, we also estimated the basal melt rate (Fig. 6b) for the temperate base area of the glacier. Within Elmer/Ice we computed the energy-balance at the bed from an estimated geothermal heat flux,
strain heating and basal friction-heating (Gong et al., 2017). Relatively high basal melt rates (> 0.005 m a⁻¹) appeared at the side walls of the sub-glacial valley around the over-deepening area, mainly caused by frictional heating. The basal melt water followed similar patterns of flow as the surface melt water that reaches the bed.

**Discussion**

Previous studies of the surge in Basin 3 (Dunse et al., 2012, 2015; Gladstone et al., 2014) revealed an atypical surge activation phase with multi-decadal acceleration superimposed, for at least 6 years, by short-lived, abrupt seasonal speed-up events that were clearly related to summer melt, which could not be explained solely by the thermal switch mechanism (Murray et al., 2003) typical of polythermal surging glaciers in Svalbard.

We used the discrete element model – HiDEM (Åström et al., 2014) to locate the possible location of crevasses. In general the modeled crevasses distribution in August 2013 matches with the crevasses map derived from satellite observation. However there is a mismatch of the orientation in the middle upper area (Fig. 5c). It may be due to that HiDEM only simulates the ad-hoc formation but not the advection of crevasses, thus no crevasse formation history can be inferred from the model. The inclusion of crevasse advection could be implemented in a two-way coupling of HiDEM with a continuum model accounting for damage transport in future studies. The mismatch of the crevasse density (Fig. 5c) at the northern and southern frontal area could be caused by the mismatch of ice front position between reality and the model. Although in reality the ice front advanced for several kilometers after the full-surge, it was kept fixed in position in Elmer/Ice (Sect. 3.1). The shape and steepness of the ice front likely affects the behavior of the discrete element model. However, as they are concentrated at the terminus of the glacier, these crevasses are less likely to affect the basal hydrology system on a wider scale.

We then selected the modeled crevasses in August 2012 that may penetrate ice deep enough to act as routing paths of surface melt water to the bed. In this study we focused on the cut-through crevasses formed above the margins of the sub-glacial valley because the basal flow pattern of the surface melt entering through those crevasses was indicative of potential subglacial water routing and hydrology.

Secondly, some of the basal water flow paths presented in Fig. 6a and 6b terminate under a plateau in the hydraulic potential which occurs in the over-deepened bedrock region (see also Fig. 1b). Given the very low gradients of our calculated hydraulic potential in this region and the presence of a local hydraulic potential minimum slightly downstream of the over-deepening, basal water would likely have low flow speeds, and possibly even accumulate in the over-deepened bedrock region, over time. This may have impacted on the surge development in Basin 3. Also given that the lowest basal resistance during most of 2012 (Fig. 4a) was immediately downstream of the over-deepening area in the northern flow unit, outflow of accumulated water...
likely enhanced the surge activation here. If seasonal surface melt water accumulates here and drains over a longer period, this may explain prolonged high ice velocities after the melt season has ended. The temporary speed-up of the southern flow unit in 2008 (Gladstone et al., 2014) could plausibly have been triggered by an influx of basal water that was not repeated again until the basin wide surge was initiated. An outburst of basal water accumulated in the over-deepened bedrock region could provide one possible mechanism for this to occur. A ridge in hydraulic potential divides the northern and southern flow units in August 2012 (Fig. 6a). An anomalously high inflow of surface meltwater could have caused this ridge to be flooded if regular drainage channels were of insufficient capacity. We are unable to say how likely this is without a time series of surface melt data including the 2007 and 2008 seasons, but such an event could cause a temporary speed-up to the southern flow unit. Englacial channels which may cause a redistribution of water within the hydrologic system (Fountain and Walder, 1998) are not directly considered in the current study. We assume that direct transfer of surface runoff via cut-through crevasses exceeds the englacial water transport at Basin 3.

Lastly, we look at the role of basal melt water in the activation of the southern flow unit. Basal meltwater from further upstream in the northern flow unit can drain toward the southern unit (Fig 6b; prior to the basin-wide surge, nearly all of the ice drained toward the northern flow unit). If this basal meltwater accumulated upstream due to the lower part of the glacier being below pressure melting point, such accumulated basal melt water could have caused the speed-up once basal temperatures reached melting point under the southern corner and the hydrologic system extended beneath the southern flow unit. Also basal melt water generated locally in the over-deepening area (Fig. 6b) may not have been able to drain completely in one season thus could be accumulated locally. However whether basal melt water can eventually burst out from the over-deepening area and contribute to the seasonal speed-up events or refreeze locally depends also on the development of the hydrology system and the thermal regime.

Although we lack either simulated or observed surface melt volumes for summer 2012 we would expect that the surface melt is much larger than basal melt. The runoff output from HIRHAM5 regional climate model in 1995 (personal information from R. Mottram of Danish Meteorological Institute; 1995 was not a year with high surface melt) at the location of the cut though crevasses was at least 10 time larger than the basal melt rate in either 1995 or 2012. The volume of surface melt observed at weather stations located in southwestern Basin 3 after summer in 2004 was also at least 10 times larger (Schuler et al., 2007). Considering the seasonal timings and magnitudes of speed-up events, and the feedback between surface melt water input and hydraulic warming at the bed, it is clear that surface melt, when it can penetrate to the bed, causes a much higher impact on sliding and ice dynamics than basal melt water.

Then we also discuss the role of the crevasses formation in the long term acceleration. These are initiated as a consequence of extensional flow resulting from changes in the basal thermal structure in an early post-quiescent phase and act as the triggering and enhancing factor in the so-called “annual hydro-thermodynamic feedback” proposed by Dunse et al. (2015). While Dunse et al. (2015) are unspecific as to the cause of “hydro-thermodynamic” initiation zone in the long term glacier acceleration, we propose that basal melt water resulting from the gradual thickening of ice (raising basal temperatures) during the quiescent phase, could sufficiently enhance flow speeds to initiate cut-through crevassing. The basal temperature distribution inversely calculated from the glacier geometry and velocity (Gong et al., 2016) showed a partially temperate bed in 1995 and expansion of the temperate region from 1995 to 2011, which is consistent with the presence of water at the bed. Given that basal meltwater fluxes are likely to be at least an order of magnitude lower than surface meltwater or runoff fluxes.
basal melt probably has a relatively small influence on glacier sliding. We suggest that water at the bed, which is likely to be primarily routed toward the northern rather than southern flow unit due to topographic constraints (Fig. 6b), caused the speed up from the quiescent phase during the last part of the 20th century and early 21st century.

This would require two key developments from quiescent to surge phase. Firstly, the initiation of sliding after ice thickening provided sufficient insulation for the bed to reach pressure melting temperature and generate meltwater. This could have occurred during the early nineties. Then at some point before August 2012, extensional flow due to sliding became sufficient to cause cut-through crevasses leading to further acceleration and the surge onset due to the annual “hydro-thermodynamic” feedback. We have demonstrated that cut-through crevasses are likely to be present just prior to the surge in Basin 3, and that surface meltwater can flow along the paths corresponding to the regions of observed fast flow. In the end, our results support the “hydro-thermodynamic” mechanism, in which crevasses provide access for surface meltwater to reach the bed. We have demonstrated that cut-through crevasses are likely to be present approaching the surge in Basin 3, and that water flow paths route surface meltwater along flow paths corresponding to the regions of observed fast flow. While Dunse et al. (2015) are unspecific as to the cause of “hydro-thermodynamic” initiation zone, we propose that basal melt water, resulting from the build-up of the reservoir area and gradual thickening of ice (and hence raising of basal temperatures) during the quiescent phase, could sufficiently enhance flow speeds to initiate cut-through crevasses. Given that basal meltwater fluxes are likely to be at least an order of magnitude lower than surface meltwater or runoff fluxes, their impact on glacier sliding is likely to be much smaller. We suggest that basal meltwater, which is likely to be primarily routed toward the northern rather than southern flow unit due to topographic constraints (Fig. 6b), caused the speed up from the quiescent phase during the last part of the 20th century and early 21st century. This would require two key developments from quiescent to surge phase. Firstly, the initiation of sliding after ice thickening provided sufficient insulation for the bed to reach pressure melting temperature and generate sufficient meltwater, which could have occurred during the early nineties. Then at some point before August 2012 extensional flow due to sliding could have become sufficient to cause cut-through crevasses, leading to further acceleration and the surge onset due to the annual “hydro-thermodynamic” feedback.

Direct verification of the long term evolution of the surge active phase discussed above cannot be provided without quantification of the water reaching the bed and a basal sliding relation engaging the basal effective pressure. However our approach and results can throw some light on future studies of coupled ice dynamic/thermodynamic/hydrology simulations.

Conclusions

We have forced the discrete element model HiDEM with outputs from the continuum ice dynamic model Elmer/Ice to study the crevasse distribution during the surge in Basin 3, Austfonna ice-cap in 2012-2013. Our continuum to discrete multi-model approach provides simulated locations where cut-through crevasses allow surface melt water to be routed to the bed. We have demonstrated that automatic crevasse detection through Radon-transform may be used to validate simulated crevasse distribution from our continuum-discrete modelling approach. With the future addition of a basal hydrology model, the current study constitutes a step towards a fully coupled ice-dynamic englacial/basal hydrology modelling system in which both input locations of input surface water and basal meltwater generation are represented.

Our results support the “hydro-thermodynamic” feedback to summer melt proposed by Dunse et al (2015) to explain the seasonal speed-up in Basin-3 and the initiation of the acceleration of the southern flow unit in 2012. The calculated flow paths
of the basal water according to hydraulic potential indicate either a direct enhancement to the ice flow through basal lubrication or a lagged-in-time mechanism through the outflow of accumulated water in the over-deepening area.

We propose that basal melt water production caused the speed up from the quiescent phase of Basin 3 during the last part of the 20th century and early 21st century. Then the “hydro-thermodynamic” feedback initiated during 2011 or early 2012 causing the activation of the southern flow unit and the expansion of the surge across the entire basin. The quantification of the roles and mechanisms involving basal melt water production, surface melt water and crevasse opening for the surge discussed in this study need to be further improved by coupling basal hydrology with the thermal regime evolution and surface mass and energy balance.

**Author contribution.** Y. Gong and T. Zwinger designed the numerical experiments and carried out the simulation in Elmer/Ice. J. Äström carried out the simulations in HiDEM. B. Altena produced the crevasse map with Radon-transform. T. Schellenberger processed and produced the TSX velocity time series. Y. Gong analyzed the model results and designed the figures. Y. Gong wrote the manuscript together with R. Gladstone, J. Moore and T. Zwinger. All the authors assisted in data interpretation and commented on/edited the paper.

**Acknowledgements.** We wish to thank all the partners for providing data and constructive discussion during the study, especially R. Mottram from Danish Meteorological Institute for the HIRHAM5 surface mass balance; T. Strozza from GAMMA Remote Sensing and Consulting AG for the ERS-2 SAR surface velocity observation acquired in March to April 2011; M. McMillan from the University of Leeds for the surface elevation derived from Cryosat altimetry data acquired during July 2010 – December 2012; T. Dunse from the University of Oslo for the bedrock and ice thickness data and A. Kääb from the German Aerospace Center DLR (LAN_0211) for TSX data. We wish to thank F. Gillet-Chaulet from Laboratoire de Glaciologie et Géophysique de l’Environnement for making available his code for the inverse modelling. We also acknowledge CSC IT Center for Science Ltd. for the allocation of computational resources. The work was supported by Finnish Academy project 286587: Simulating Antarctic marine ice sheet stability and multi-century contributions to sea level rise. T. Zwinger is supported by the Nordic Center of Excellence eSTICC (eScience Tools for Investigating Climate Change in Northern High Latitudes) funded by Nordforsk (grant 57001). B. Altena is funded by the European Research Council under the European Union’s Seventh Framework Programme grant agreement No. 320816. T. Schellenberger was funded by the Research Council of Norway (RASTAR, 208013), the Norwegian Space Centre as part of European Space Agency’s PRODEX program (C4000106033), and the European Union FP7 ERC project ICEMASS (320816).

**Reference**


Table 1 TerraSAR-X acquisitions of Basin 3 and repeat-pass period

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<tr>
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Figure 1. Surface and bedrock topography of Basin 3, Austfonna. (a) Surface elevation of Basin 3 contours with solid black lines (with ~48.2 m interval), on top of a satellite image of Nordaustlandet from TerraColor® Global Satellite Imagery (http://www.terracolor.net/). The gray transparent box shows the coverage of the TerraSAR-X scene (30 April 2012). The model domain of HiDEM is outlined with red box. The insert at the upper left corner shows the ice cap’s location within the Svalbard archipelago. (b) Bedrock topography is color-coded, contoured with black solid line with a ~37.1 m interval and superimposed by surface elevation contours (white solid line with ~ 48.2 m interval). The gray solid line outlines Basin 3 and the model domain of Elmer/Ice in both panels.
Figure 2. Basal friction coefficient inverted from surface velocity data in (a) 18-29 August 2012 ($C_{pre}$) and (b) 16-27 August 2013 ($C_{post}$). Both panels display basal friction coefficient shown onto the left, surface velocity data after post-processing (Sect. 2.2) shown on the upper right and the relative difference between observed and modeled surface velocity magnitude shown on the lower right.
Figure 3a. The crevasse maps created from Radon-transform. (a) The orientation of crevasses indicated by a color wheel, in which the strength of the signal controls the saturation; (b) The highest responding orientation from the Radon-transform ($\hat{s}(\theta)$) with the color bar indicating the intensity; (c) The cartographic map indicating both the orientation and intensity of the two strongest responding orientations. Basin 3 is outlined by black solid line.
Figure 4. The evolution of basal friction coefficient ($C$), with the corresponding observed speeds plotted below, shown for the model domain of HiDEM. 

(a) $\log_{10}(C)$, overlain with white contour lines showing $\log_{10}(C) = -3.5$ (low friction), from the time before the peak of the surge. 

(b) $\log_{10}(C)$, overlain with black contour lines showing $\log_{10}(C) = -5.5$ (almost vanishing friction), from the time period at and after the peak of the surge. The speed in 2011 was acquired from ERS-2 SAR imagery (color bar on the left). The rest of the speeds snapshot were from TSX SAR (color bar on the right). The grey solid line outlines the Basin 3 boundary.
Figure 5. Crevasse distribution from HiDEM on (a) August 2012 and (b) August 2013 and (c) satellite observation. The color of the underlying image in (a) and (b) shows the surface elevation of the glacier. Bedrock topography contours are shown in black with a ~23.7 m interval. All the dots in both (a) and (b), regardless of the color, indicate the modeled crevasse distribution from HiDEM. The red dots are the cut-through crevasses. The red dots in the yellow boxes in (a) are the ones referred as cut-through crevasses above the sub-glacial valley margins and are used for calculating the flow paths of the surface melt reached the bed. The black dots in (b) (upper left and lower left corner) mark crevasses produced due to boundary effects in the model (Sect. 4.2). They are eliminated from the crevasse map. The rest of the crevasses are marked with white dots, and are mostly shallow crevasses, hence irrelevant to water routing. The cartographic representation of the observed crevasse orientation on 8 August 2013 is shown in (c) (color-coded with detecting intensity in the background). The magenta color shows the area where modeled and observed crevasse match. The basin side boundary is outlined with gray dashed line in all the sub-plots.
Figure 6. The flow paths of different water sources derived from the results in August 2012. (a) The white lines indicate the path of surface melt water after entering the basal hydrology system via cut-through crevasses (red dots) according to hydraulic potential. The modeled basal velocity magnitude is color-coded in the background. (b) The white lines indicate the water path of basal melt water from locations with in-situ melt rates above 0.005 m a⁻¹. The bedrock elevation is color-coded in the background. (c) The logarithm (base 10) of the basal melt rate is color-coded in the background. The colored contour lines indicate the value of the basal melt rate. The black contour lines in both (a) and (b) indicate the hydraulic potential.